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THESIS

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ALGORITHMS FOR COMPUTER AIDED
DESIGN OF DIGITAL FILTERS

by

Thomas H. Rich

'''

December 1988

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Robert Strum

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ALGORITHMS FOR COMPUTER AIDED
DESIGN OF DIGITAL FILTERS

by

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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

The algorithms necessary for the computer aided design of digital filters from lowpass prototypes have been developed and compared, then implemented in a computer program to aid in the instruction of digital filter design.

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I. DEVELOPMENT OF GENERAL ALGORITHMS

A. INTRODUCTION

The procedure for the design of lowpass, highpass, bandpass, and bandstop digital filters from lowpass prototypes is well established, and is based on transforming the prototype frequency response to meet desired characteristics. This transformation can be accomplished in either the analog (s) domain or the digital (z) domain (Figure 1).

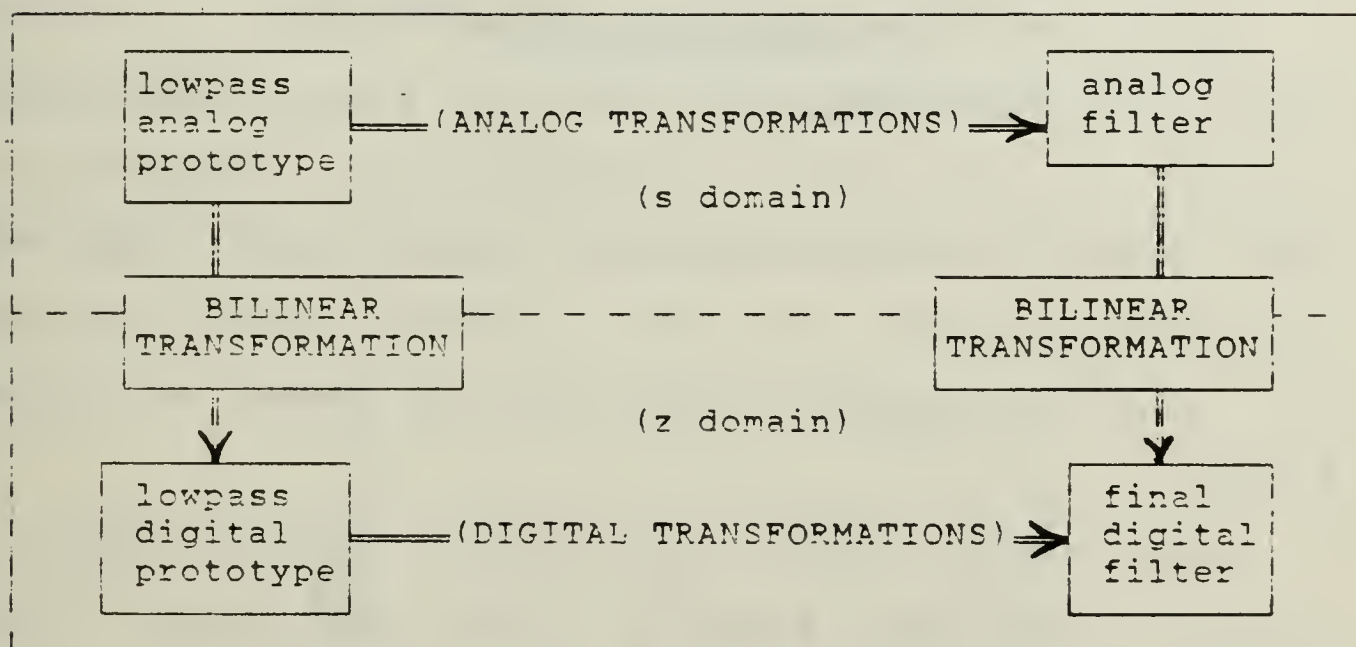


Figure 1. Two methods for digital filter design. Note that each of the horizontal arrows actually represent four different transformations:
1) lowpass prototype to lowpass filter
2) lowpass prototype to highpass filter
3) lowpass prototype to bandpass filter
4) lowpass prototype to bandstop filter

Therefore, the two classical design techniques, analog-digital and digital-digital design, are based on doing the frequency response transformations in their respective analog or digital domains. Since the theory for the standard filter types (Butterworth, Chebyshev, and elliptic) is developed in the analog or 's' domain, the two methods for designing digital filters differ basically in the sequence of the design steps. In the analog-digital method the analog prototype is transformed and then digitized, while in the digital-digital method the analog prototype is digitized and then transformed.

Each of the transformations (arrows in Figure 1) involves substituting a function of 's' or 'z' for the appropriate variable in the filter transfer function. The purpose of this thesis is to develop efficient algorithms to carry out these transformations and then to implement these algorithms in a computer program to aid in the design of digital filters.

Computer-aided design of digital filters offers a significant advantage since it allows the digital filter coefficients to be calculated quickly and accurately. The accuracy of the filter coefficients is an especially significant point since the frequency response of the final filter is very sensitive to small inaccuracies in the calculated coefficients. As an example, compare the frequency response for the filters in Figure 2 and 3. Figure

2 shows the frequency response magnitude for a 1/2-dB ripple Chebyshev bandpass filter, and Figure 3 shows the frequency response when one of the coefficients has been changed by 1/1000th.

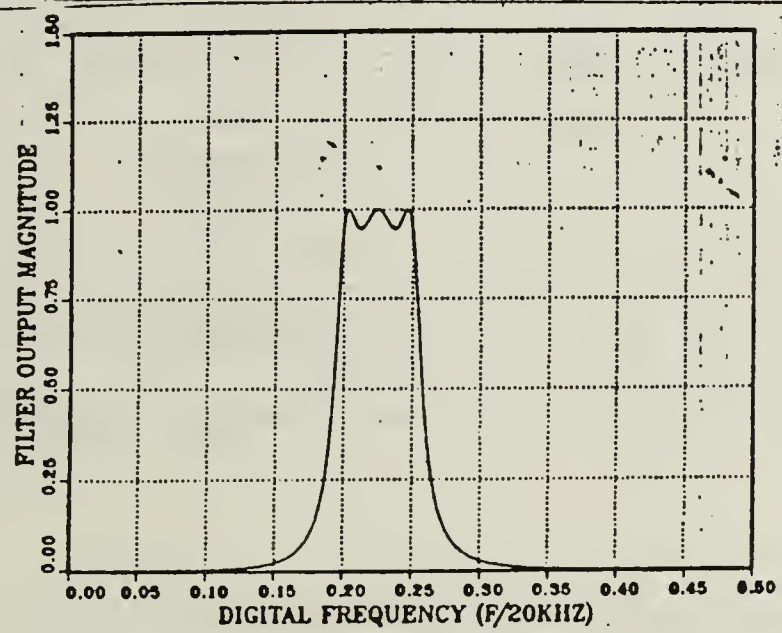


Figure 2. Chebyshev Bandpass Filter

$$H(z) = \frac{.00229(z^4 - 1)^3}{z^6 - .87775z^5 + 2.80076z^4 - 1.5389z^3 + 2.46277z^2 - .67494z + .67529}$$

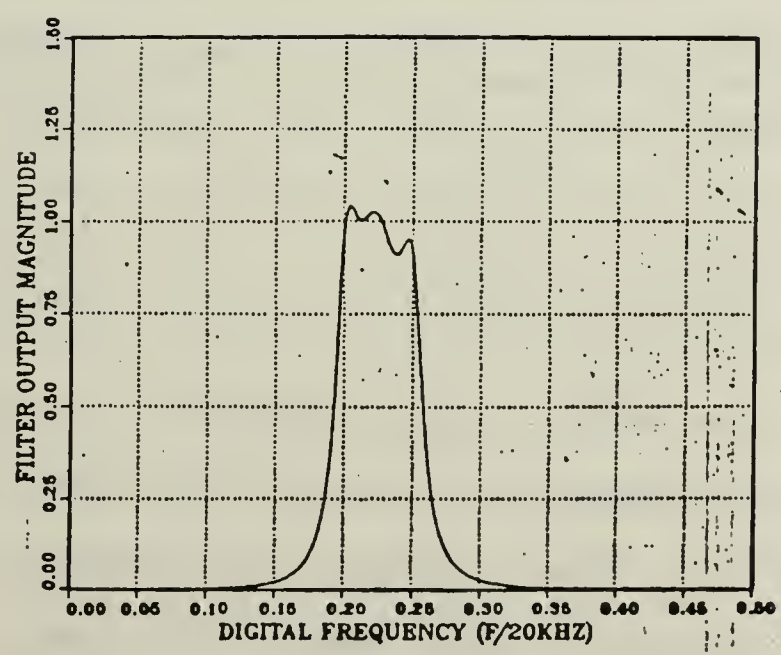


Figure 3. Same Filter With Denominator Coefficient of z changed to $-.67594$

B. GENERAL DISCUSSION

Each of the transformations involves substituting a ratio of two polynomials of s or z into the filter transfer function which is also a ratio of two polynomials in s or z , namely

$$H(y) = \frac{Y(y)}{U(y)} \quad \Bigg|_{y = \frac{p(x)}{q(x)}} \quad (1.1)$$

The variables ' x ' and ' y ' can be either ' s ' or ' z ' depending on the transformation to be performed. Therefore, we have either

$$H(y) = \frac{a_0 + a_1 y + a_2 y^2 + \dots + a_m y^m}{b_0 + b_1 y + b_2 y^2 + \dots + b_n y^n} \quad \Bigg|_{y = \frac{p(x)}{q(x)}} \quad (1.2)$$

or

$$H(x) = \frac{a_0 + a_1 \frac{p(x)}{q(x)} + a_2 \frac{(p(x))^2}{(q(x))^2} + \dots + a_m \frac{(p(x))^m}{(q(x))^m}}{b_0 + b_1 \frac{p(x)}{q(x)} + b_1 \frac{(p(x))^2}{(q(x))^2} + \dots + b_n \frac{(p(x))^n}{(q(x))^n}} \quad (1.3)$$

In general, the order of the numerator ' m ', is less than the order of the denominator ' n ' (except for the transformations in the digital domain where the result of the bilinear transformation is to make $n=m$). We can generalize the

results, however, by assuming that the order of the numerator polynomial $Y(x)$ is equal to the order of the denominator polynomial $U(x)$ and making the additional coefficients $a_{m+1}, a_{m+2}, \dots, a_n$ equal to zero. Then multiplying both the numerator and denominator of the above equation by $(g(x))^n$ gives

$$H(x) = \frac{a_0 (g(x))^n + a_1 p(x) (g(x))^{n-1} + \dots + a_n (p(x))^n}{b_0 (g(x))^n + b_1 p(x) (g(x))^{n-1} + \dots + b_n (p(x))^n} \quad (1.4)$$

or, in a different form,

$$H(x) = \frac{\sum_{i=0}^{i=n} a_i p(x)^i g(x)^{n-i}}{\sum_{i=0}^{i=n} b_i p(x)^i g(x)^{n-i}} \quad (1.5)$$

Now with the numerator and denominator of the same form, we can use the same transformation for both the numerator and denominator polynomials of the filter transfer function, namely,

$$a'(x) = a_0' + a_1'x + a_2'x^2 + \dots + a_n'x^n = \sum_{i=0}^{i=n} a_i p(x)^i g(x)^{n-i} \quad (1.6)$$

In Equation 1.6, the a_i are the coefficients of the old numerator or denominator polynomials and a_i' are the new, transformed numerator or denominator polynomial coefficients. Since the new coefficients a_i' are a linear combination of

the old coefficients a_i , and the coefficients of $p(x)$ and $q(x)$, this suggests the use of a matrix transformation;

$$\mathbf{a}' = \mathbf{a} \mathbf{A}(n) \quad (1.7)$$

where

\mathbf{a} is the coefficient row vector for the old numerator or denominator polynomial of length $= n+1$.

$$\mathbf{a} = [a_0 \ a_1 \ a_2 \ \dots \ a_n]$$

\mathbf{a}' is the coefficient row vector for the new numerator or denominator polynomial of length $= n'+1$.

$$\mathbf{a}' = [a'_0 \ a'_1 \ a'_2 \ \dots \ a'_n]$$

$\mathbf{A}(n)$ is the transformation matrix with dimensions $(n+1) \times (n'+1)$. The elements of $\mathbf{A}(n)$ depend upon the coefficients of $p(x)$ and $q(x)$. Note that if the rows and columns of $\mathbf{A}(n)$ are numbered 0 to n' , then

$$a'_j = \sum_{i=0}^{i=n} a_i A(n)_{ji} \quad (1.8)$$

(where $A(n)_{ji}$ denotes the element in row i , column j of matrix $\mathbf{A}(n)$.)

n is the order of the transfer function before the transformation

n' is the order of the transfer function after the transformation. Note that n' equals n times the highest order of $p(x)$ and $q(x)$.

For example, when $n=1$ and $n'=2$, $p(x)$ and $q(x)$ are second order and Equation 1.7 becomes,

$$\begin{bmatrix} a_0 & a_1 \end{bmatrix} \begin{bmatrix} A(2)_{00} & A(2)_{01} & A(2)_{02} \\ A(2)_{10} & A(2)_{11} & A(2)_{12} \end{bmatrix} = \begin{bmatrix} a'_0 & a'_1 & a'_2 \end{bmatrix} \quad (1.9)$$

In the case of the lowpass digital, highpass digital, and bilinear transformations, the two functions $p(x)$ and $g(x)$ are first order. Professor P.H. Moose of the Naval Postgraduate School of Monterey has developed the matrix $A(n)$ for these two cases by expanding Equation 1.6 using the binomial theorem (Reference 1). This method cannot be used for transformations involving higher order polynomials such as the bandpass and bandstop transformations and it is difficult to develop because intricate manipulation of the binomial coefficients are involved. The binomial expansion approach, therefore, was not pursued.

A general method has been found that develops the required transformation matrices in an iterative manner. In other words, the matrix used to transform a numerator or denominator filter polynomial of order n is developed from the matrix for the polynomial of order $n-1$. This approach is easy to implement in a computer program and can be used for any of the polynomial substitutions needed. The next chapter is concerned with developing the transformation matrices for each specific transformation. However, as an introduction to the method used, consider the generic case where both $p(x)$ and $g(x)$ are second-order polynomials. Since all of the analog and digital transformations involve substitutions with polynomials of second order or less, this second-order generic case will be all that is needed to develop the specific transformations later.

With $p(x)=ax^2 + bx + c$, $q(x) = dx^2+ ex + f$, and substituting into Equation 1.6 for the first order case of $n=1$ (remembering that with $p(x)$ and $q(x)$ of second order, the resulting polynomial will be second order), we get

$$a'(x) = a_0' + a_1'x + a_2'x^2 = a_0(dx^2 + ex + f) + a_1(ax^2+bx+c) \quad (1.10)$$

Using Equation 1.8 or 1.9 it is easy to pick out the matrix coefficients $A(1)_{ij}$ ($i=0,1$; $j=0,1,2$) for the first order generic case. The coefficients are,

$$A(1) = \begin{bmatrix} f & e & d \\ c & b & a \end{bmatrix} \quad (1.11)$$

Now with $n=2$ ($n'=4$) substituting into Equation 1.6 gives,

$$\begin{aligned} a'(x) &= a_0' + a_1'x + a_2'x^2 + a_3'x^3 + a_4'x^4 \\ &= a_0(dx^2+ex+f)^2 + a_1(ax^2+bx+c)(dx^2+ex+f) + a_2(ax^2+bx+c)^2 \end{aligned} \quad (1.12)$$

and the matrix $A(2)$ is;

$$A(2) = \begin{bmatrix} f^2 & 2ef & 2df+e^2 & 2de & d^2 \\ fc & ec+bf & cd+fa+eb & db+ea & da \\ c^2 & 2bc & 2ac+b^2 & 2ab & a \end{bmatrix} \quad (1.13)$$

To see how this matrix is derived from the previous one, consider the first row of the matrices in Equations 1.11 and 1.13. For both matrices, this row is multiplied by the a_0 coefficient in the old polynomial. For $n=1$ we have $a_0 p(x)$

and we pick out the coefficients of the powers of x (x') to get the matrix elements $A(n)_{ij}$. For $n=2$, we have $a_0(p(x))^2$ and we do the same thing only now we have the additional factor of $p(x)$. The matrix elements can be calculated by hand in this fashion but it requires a lot of algebra. A simpler method is to use the matrix elements of the previous matrix and get the appropriate elements by convolving coefficients. For example, consider the sequence of elements in row 0 (the top row) of $A(1)$, namely

$$f \quad e \quad d \quad (1.14)$$

To get the elements for row 0 of $A(2)$, we need to multiply (convolve coefficients) by another $p(x)$. Therefore, with rows numbered 0 to n , and columns number 0 to n' , we have the following.

For row 0 column 0, the appropriate matrix element is

$$A(2)_{00} = \begin{matrix} d & e & f \\ f & e & d \end{matrix} = f^2 \quad (1.15)$$

For row 0 column 1, the appropriate matrix element is

$$A(2)_{01} = \begin{matrix} d & e & f \\ f & e & d \end{matrix} = 2fe \quad (1.16)$$

For row 0 column 2, the appropriate matrix element is

$$A(2)_{02} = \begin{matrix} d & e & f \\ f & e & d \end{matrix} = 2fd + e^2 \quad (1.17)$$

For row 0 column 3, the appropriate matrix element is

$$A(2)_{03} = \begin{matrix} & d & e & f \\ f & e & d & \end{matrix} = 2de \quad (1.18)$$

For row 0 column 4, the appropriate matrix element is

$$A(2)_{04} = \begin{matrix} & d & e & f \\ f & e & d & \end{matrix} = d^2 \quad (1.19)$$

This convolution of the coefficients of the old matrix with the coefficients of $p(x)$ can be carried out for every row of the old matrix to obtain all the rows of the new matrix except the last row. In the case of the last row, we are multiplying by an additional $g(x)$ rather than an additional $p(x)$ and so must obtain the last row of the new matrix by convolving the coefficients of the last row of the old matrix with the coefficients of $g(x)$.

In summary we have,

(i) for row 0 to row $n-1$:

$$\text{-row } i \text{ of } A(n) = \text{row } i \text{ of } A(n-1) * \text{coefficients of } p(x) \quad (1.20)$$

(ii) for row n :

$$\text{-row } n \text{ of } A(n) = \text{row } n-1 \text{ of } A(n-1) * \text{coefficients of } g(x) \quad (1.21)$$

where $*$ indicates linear convolution of the coefficients.

To see how this method of convolving coefficients works, consider multiplying two polynomials:

$$f1(x) = f2(x)f3(x) \quad (1.22)$$

Since 'x' is the independent variable, no matter what letter we use, the polynomials will remain the same. Substituting 'z⁻¹' for 'x' in the above equation gives us,

$$f_1(z^{-1}) = f_2(z^{-1})f_3(z^{-1}) \quad (1.23)$$

and we now have the familiar form where we can either multiply the two functions of 'z' or convolve the sequences found by taking the inverse z-transform. For f₂(z⁻¹) and f₃(z⁻¹) we find

$$\begin{aligned} f_2(z^{-1}) &= a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n} \\ f_3(z^{-1}) &= b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_m z^{-m} \end{aligned} \quad (1.24)$$

It is now clear that we can find the coefficients of the product f₁(x) by convolving the sequence {a_n} with the sequence {b_m} to produce the sequence {c_q} where,

$$\{a_n\} = \{a_0, a_1, a_2, a_3, \dots, a_n\}$$

$$\{b_m\} = \{b_0, b_1, b_2, b_3, \dots, b_m\}$$

$$\text{and with } q = m + n \quad (1.25)$$

$$\{c_q\} = \{c_0, c_1, c_2, c_3, \dots, c_q\}$$

Then by taking the z-transform of the sequence {c_q} and substituting 'x' for 'z⁻¹' we get,

$$f_1(x) = c_0 + c_1 x + \dots + c_q x^q \quad (1.26)$$

We now have a simple straight-forward method of developing the necessary transformation matrices for digital filter design by picking out the first-order transformation matrix using Equations 1.6 and 1.11, and then using the convolution of coefficients method iteratively on the rows of each successive transformation matrix with Equations 1.20 and 1.21 until the appropriate matrix $A(n)$ has been generated. The next chapter will develop the transformation matrices for each of the following transformations:

- bilinear transformation
- digital lowpass prototype to digital lowpass filter
- digital lowpass prototype to digital highpass filter
- digital lowpass prototype to digital bandpass filter
- digital lowpass prototype to digital bandstop filter
- analog lowpass prototype to analog lowpass filter
- analog lowpass prototype to analog highpass filter
- analog lowpass prototype to analog bandpass filter
- analog lowpass prototype to analog bandstop filter.

We will see that some of these transformations are similar and consequently, we will not have to develop separate transformation matrices for each case.

II. DEVELOPMENT OF SPECIFIC TRANSFORMATIONS

A. INTRODUCTION

The individual transformations are discussed in the following sections. The specific form of the transformations used in each case are from Reference 2. Since the final result will be to develop a computer program to assist in the calculation of digital filter coefficients, confining the specific transformations to the form in Reference 2, will allow the computer program to be used in conjunction with Reference 2 as an aid in designing digital filters.

B. BILINEAR TRANSFORMATION

The bilinear transformation may be used to transform an analog transfer function, $H(s)$, to a digital transfer function $H(z)$. The transformation from $H(s)$ to $H(z)$ is described by

$$H(z) = H(s) \Big|_{s = \frac{z-1}{z+1}} \quad (2.1)$$

Referring to Equation 1.1, $p(x)=z-1$ and $q(x)=z+1$. Since both of the transformation polynomials $p(x)$ and $q(x)$ are first order, the bilinear transformation will not change the order of the transfer function. Therefore, for an analog transfer function of order 'n', the transformation matrix $A(n)$ will be

(n+1) by (n+1). From Equation 1.6, the numerator or denominator polynomial of the transfer function $H(z)$ is given by

$$a'(z) = a_0' + a_1'z + \dots + a_n'z^n = \sum_{i=0}^{i=n} a_i (z-1)^i (z+1)^{n-i} \quad (2.2)$$

where the unprimed coefficients, a_i , come from the analog transfer function's numerator or denominator polynomial. In matrix form, with $n=1$ and referring to Equation 1.11, the first order matrix $A(1)$ for the bilinear transformation is

$$A(1) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \quad (2.3)$$

Using the convolution of coefficients method developed in Chapter 1, the transformation matrix $A(n+1)$ can be developed from the matrix $A(n)$. The results for the bilinear transformation matrices $A(1)$ through $A(5)$ are given in Table 1.

TABLE 1
FIRST-ORDER TO FIFTH-ORDER BILINEAR TRANSFORMATION MATRICES

$$A(1) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$A(2) = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$A(3) = \begin{bmatrix} 1 & 3 & 3 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

$$A(4) = \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ -1 & -2 & 0 & 2 & 1 \\ 1 & 0 & -2 & 0 & 1 \\ -1 & 2 & 0 & -2 & 1 \\ 1 & -4 & 6 & -4 & 1 \end{bmatrix}$$

$$A(5) = \begin{bmatrix} 1 & 5 & 10 & 10 & 5 & 1 \\ -1 & -3 & -2 & 2 & 3 & 1 \\ 1 & 1 & -2 & -2 & 1 & 1 \\ -1 & 1 & 2 & -2 & -1 & 1 \\ 1 & -3 & 2 & 2 & -3 & 1 \\ -1 & 5 & -10 & 10 & -5 & 1 \end{bmatrix}$$

Referring to Table 1 and keeping in mind the convolution of coefficients method that produced the matrices, the following relations are apparent and will be useful in computer implementation.

$$A(n)_{0,j} \text{ (the first row)} = \binom{n}{j} \quad \text{for } j=0,1,\dots,n \quad (2.4)$$

where $\binom{n}{j}$ indicates the binomial coefficient

$$A(n)_{i,0} \text{ (the first column)} = (-1)^i \quad \text{for } i=0,1,\dots,n \quad (2.5)$$

$$A(n)_{i,n} \text{ (the last column)} = 1 \quad \text{for } i=0,1,\dots,n \quad (2.6)$$

$$A(n)_{(n-i),i} = (-1)^{n-j} A(n)_{i,i} \quad \text{for } i=0,1,\dots,\text{integer}(n/2)+1 \quad (2.7)$$

$$A(n)_{i,j} = A(n-1)_{i,j} + A(n-1)_{i,j-1} \quad \text{for } i=0,1,\dots,n-1 \quad (2.8)$$

$$j=1,2,\dots,n-1$$

Notice that the values of the matrix elements for the bilinear transformation are whole numbers. This is important from a numerical accuracy standpoint since whole numbers can be expressed precisely in floating point form. This means that the bilinear transformation will cause a loss of accuracy of the original coefficients only because of the $n+1$ multiplications and additions used to calculate each new digital coefficient.

C. LOWPASS TO LOWPASS DIGITAL TRANSFORMATION

The lowpass to lowpass digital transformation is used to transform a lowpass prototype digital filter $H(z)$ into a lowpass digital filter $H(z)$. The polynomial substitution

used is

$$H(z) = H_p(z) \bigg|_{z = \frac{z - \alpha}{1 - \alpha z}} \quad (2.9)$$

where the subscript p indicates the prototype digital filter transfer function. The design constant α is determined [References 2 and 3] by

$$\alpha = \frac{\sin(\theta_c/2 - \theta_c'/2)}{\sin(\theta_c/2 - \theta_c''/2)} \quad (2.10)$$

with θ_c = the prototype critical frequency (usually $\pi/2$) and θ_c' = the desired critical frequency of the lowpass filter. Again, referring to Equation 1.1 with $p(x) = z - \alpha$ and $g(z) = 1 - \alpha z$, the lowpass digital transformation matrix will not change the order of the transfer function and the transformation matrix $A(n)$ will be $(n+1)$ by $(n+1)$. Using Equations 1.6 and 1.11 as before, the first order transformation matrix $A(1)$ for the lowpass digital transformation is:

$$A(1) = \begin{bmatrix} 1 & -\alpha \\ -\alpha & 1 \end{bmatrix} \quad (2.11)$$

Once again using the convolution of coefficients method, the higher-order transformation matrixes $A(n)$ are developed from the lower-order matrix $A(n-1)$. The results are shown in Table 2 for the first-order to fifth-order transformation matrices.

TABLE 2
FIRST-ORDER TO FIFTH-ORDER LOWPASS DIGITAL
TRANSFORMATION MATRICES

$$A(1) = \begin{bmatrix} 1 & -\alpha \\ -\alpha & 1 \end{bmatrix}$$

$$A(2) = \begin{bmatrix} 1 & -2\alpha & \alpha^2 \\ -\alpha & (1+\alpha^2) & -\alpha \\ \alpha^2 & -2\alpha & 1 \end{bmatrix}$$

$$A(3) = \begin{bmatrix} 1 & -3\alpha & 3\alpha^2 & -\alpha^3 \\ -\alpha & (1+2\alpha^2) & (-2\alpha-\alpha^3) & \alpha^2 \\ \alpha^2 & (-2\alpha-\alpha^3) & (1+2\alpha^2) & -\alpha \\ -\alpha^3 & 3\alpha^2 & -3\alpha & 1 \end{bmatrix}$$

$$A(4) = \begin{bmatrix} 1 & -4\alpha & 6\alpha^2 & -4\alpha^3 & \alpha^4 \\ -\alpha & (1+3\alpha^2) & (-3\alpha-3\alpha^3) & (3\alpha^2+\alpha^4) & -\alpha^3 \\ \alpha^2 & (-2\alpha-2\alpha^3) & (1+4\alpha^2+\alpha^4) & (-2\alpha-2\alpha^3) & \alpha^2 \\ -\alpha^3 & (3\alpha^2+\alpha^4) & (-3\alpha-3\alpha^3) & (1+3\alpha^2) & -\alpha \\ \alpha^4 & -4\alpha^3 & 6\alpha^2 & -4\alpha & 1 \end{bmatrix}$$

$$A(5) = \begin{bmatrix} 1 & -5\alpha & 10\alpha^2 & -10\alpha^3 & 5\alpha^4 & -\alpha^5 \\ -\alpha & (1+3\alpha^2) & (-4\alpha-6\alpha^3) & (6\alpha^2+\alpha^4) & (-4\alpha^3-\alpha^5) & \alpha^4 \\ \alpha^2 & (-2\alpha-3\alpha^3) & (1+6\alpha^2+3\alpha^4) & (-3\alpha-6\alpha^3-\alpha^5) & (3\alpha^2+2\alpha^4) & -\alpha^3 \\ -\alpha^3 & (3\alpha^2+2\alpha^4) & (-3\alpha-6\alpha^2-\alpha^5) & (1+6\alpha^2+3\alpha^4) & (-2\alpha-3\alpha^3) & \alpha^2 \\ \alpha^4 & (-4\alpha^3-\alpha^5) & (6\alpha^2+4\alpha^4) & (-4\alpha-6\alpha^3) & (1+4\alpha^2) & -\alpha \\ -\alpha^5 & 5\alpha^4 & -10\alpha^3 & 10\alpha^2 & -5\alpha & 1 \end{bmatrix}$$

Referring to Table 2, the following relations are observed and will be used in the computer implementation.

$$A(n)_{0j} \quad (\text{the first row}) = \binom{n}{j} (-\alpha)^j \quad \text{for } j=0,1,\dots,n \quad (2.12)$$

$$A(n)_{i0} \quad (\text{the first column}) = (-\alpha)^i \quad \text{for } i=0,1,\dots,n \quad (2.13)$$

$$A(n)_{ij} = A(n)(n-i)(n-j) \quad \text{for } i,j=0,1,\dots,n \quad (2.14)$$

$$A(n)_{ij} = A(n-1)(i-1)(j-1) + (-\alpha)A(n-1)(i-1,j) \quad (2.15)$$

for $i=1,2,\dots,n-1$
 $j=1,2,\dots,n$

Note that the elements of the lowpass digital transformation matrices are not whole numbers as in the bilinear transformation case, but are polynomials of the design constant α . Numerical error is introduced in this case by both the $n+1$ additions or multiplications, and the error in calculating the matrix elements that multiply the original coefficients.

D. LOWPASS TO HIGHPASS DIGITAL TRANSFORMATION

The lowpass to highpass digital transformation is used to transform a lowpass prototype to a highpass filter. The polynomial substitution for this transformation is:

$$H(z) = H_p(z) \Big|_{z = -\frac{z-\alpha}{1-\alpha z}} \quad (2.16)$$

Since this transformation is the same as that for the lowpass to lowpass digital transformation except for the minus sign,

we can make the substitution of '-z' for 'z' in the original prototype transfer function and then use the same transformation matrices as the lowpass to lowpass case. For the lowpass to highpass transformation, the design constant α is (References 2 and 3)

$$\alpha = \frac{\cos(\theta_c/2 - \theta_c'/2)}{\cos(\theta_c/2 + \theta_c'/2)} \quad (2.17)$$

with θ_c - the prototype critical frequency (usually $\pi/2$) and θ_c' - the desired critical frequency of the highpass filter.

E. LOWPASS TO BANDSTOP DIGITAL TRANSFORMATION

The polynomial substitution which transforms a digital lowpass prototype to a bandstop digital filter is (References 2 and 3):

$$H(z) = H_p(z) \left| \begin{array}{l} z^2 - \frac{2\alpha}{1+k} z + \frac{1-k}{1+k} \\ z = \frac{1 - \frac{2\alpha}{1+k} z + \frac{1-k}{1+k} z^2}{1} \end{array} \right. \quad (2.18)$$

with

$$\alpha = \frac{\cos(\theta_u'/2 + \theta_l'/2)}{\cos(\theta_u'/2 - \theta_l'/2)} \quad (2.19)$$

and

$$K = \tan(\theta_c/2) \tan(\theta_u'/2 - \theta_l'/2) \quad (2.20)$$

with once again, θ_c = the prototype critical frequency (usually $\pi/2$), θ_u' = the desired upper critical digital

frequency, and Θ_1' = the desired low critical digital frequency.

By letting $b = 2\alpha/(k+1)$ and $c = (1-k)/(1+k)$, Equation 2.18 can be simplified to

$$H(z) = H_p(z) \left| z = \frac{z^2 - bz + c}{1 - bz + cz^2} \right. \quad (2.21)$$

Again, using Equations 1.6 and 1.11 the first-order transformation matrix is

$$A(1) = \begin{bmatrix} 1 & -b & c \\ c & -b & 1 \end{bmatrix} \quad (2.22)$$

Note that since the substitution polynomials are second order, this transformation will double the order of the original prototype filter. Therefore, the transformation matrix $A(n)$ will be $(n+1)$ by $(2n+1)$. The higher order transformation matrices are again developed recursively and the results for the first-order to third-order transformations are given in Table 3.

TABLE 3
FIRST-ORDER TO THIRD-ORDER BANDSTOP
DIGITAL TRANSFORMATION MATRICES

$$A(1) = \begin{bmatrix} 1 & -b & c \\ c & -b & 1 \end{bmatrix}$$

$$A(2) = \begin{bmatrix} 1 & -2b & (b^2+2c) & -2bc & c^2 \\ c & (-b-bc) & (1+b^2+c^2) & (-b-bc) & c \\ c^2 & -2bc & (b^2+2c) & -2c & 1 \end{bmatrix}$$

$$A(3) = \begin{bmatrix} 1 & -3c & (3b^2+3c) & (-b^3-6bc) & \dots \\ c & (-2bc-b) & (1+2b^2+b_c+2c) & (-b^3-2b-2bc-2bc^2) & \dots \\ c^2 & (-2bc-bc^2) & (c^3+2b^2c+2c+b^2) & \dots & \\ c^3 & -3c^2b & (3b^2c+3c^2) & \dots & \end{bmatrix}$$

(Redundant entries not shown for A(3) due to space limitations. Entries not shown can be found using Equation 2.25)

Notice the increased amount of calculation needed to determine the matrix elements for the bandstop case. Each matrix element is a function of 'b' and 'c' which are themselves functions of the design constants α and k . Table 3 only gives the transformation matrices for first- to third-order. However, to illustrate the amount of calculation required, the matrix element $A(5)_{25}$ is

$$A(5)_{25} = (-12bc^2 - 6bc^3 - 8b^3c - 6bc - 6b^3 - 3b - 3bc^4 - 6b^3c^2 - b^5) \quad (2.23)$$

Since each coefficient in the final filter is found by multiplying $n+1$ matrix elements by the $n+1$ original

coefficients and then adding the $n+1$ products, the accuracy of the design constants and the original coefficients can be critical in the calculation of the final filter coefficients.

The symmetry relations for the bandstop transformation are:

$$A(n)_{i0} \quad (\text{the first column}) = c^i \quad \text{for } i=0,1,\dots,n \quad (2.24)$$

$$A(n)_{ij} = A(n)_{(n-i)(2n-j)} \quad \text{for } i=0,1,\dots,n \quad (2.25) \\ j=0,1,\dots,2n$$

$$A(n)_{ij} = A(n-1)_{ij} - bA(n-1)_{i(j-1)} + cA(n-1)_{i(j-2)} \quad (2.26) \\ \text{for } i=0,1,\dots,n-1 \\ j=0,1,\dots,2n$$

F. LOWPASS TO BANDPASS DIGITAL TRANSFORMATION

Again, the polynomial substitution which transforms the lowpass prototype to a bandpass filter is

$$H(z) = H_p(z) \left| \begin{array}{l} z^2 - \frac{2\alpha k}{k+1} z + \frac{k-1}{k+1} \\ z = - \frac{1 - \frac{2\alpha k}{k+1} z + \frac{k-1}{k+1} z^2}{\phantom{z^2 - \frac{2\alpha k}{k+1} z + \frac{k-1}{k+1}}} \end{array} \right. \quad (2.27)$$

where in this case $k = \tan(\theta/2)\cot(\theta_u'/2 - \theta_l'/2)$ and α is found from the same expression as the bandstop case. With the simplification of $b=2\alpha k/(k+1)$ and $c=(k-1)/(k+1)$, Equation 2.27 becomes

$$H(z) = H_p(z) \left| \begin{array}{l} z = -\frac{z^2 - bz + c}{1 - bz + cz^2} \end{array} \right. \quad (2.28)$$

Since this is the same form as the bandstop transformation, we can use the matrices previously developed if '-z' is first substituted for 'z' as was done in the lowpass and highpass situation.

G. LOWPASS TO LOWPASS ANALOG TRANSFORMATION

The lowpass to lowpass analog transformation is used to transform an analog lowpass prototype to a lowpass analog filter. The polynomial substitution is a simple one, namely

$$H(s) = H_p(s) \left| \begin{array}{l} s = s/w_c \end{array} \right. \quad (2.29)$$

where w_c is the prewarped critical frequency ($w_c = \tan(\theta_c/2)$) and θ_c is the desired digital critical frequency ($\theta_c = 2(\pi)f_c/f_s$). This transformation matrix is found to be

$$A(n) = \begin{bmatrix} 1 & 0 & 0 & . & . & . & 0 \\ 0 & 1/w_c & 0 & . & . & . & 0 \\ 0 & 0 & 1/w_c^2 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 1/w_c^n \end{bmatrix}$$

or $A(n) = \text{diag}(1, 1/w_c, 1/w_c^2, ., ., ., 1/w_c^n)$ throughout. (2.30)

Note that in this case, each new filter coefficient depends only on multiplying the old coefficient of s' by $(w_c)^{-1}$. Therefore, any loss of accuracy due to the transformation is due to this one multiplication and the error introduced in calculation of the matrix element $(w_c)^{-1}$.

H. LOWPASS TO HIGHPASS ANALOG TRANSFORMATION

The polynomial substitution for the lowpass to highpass transformation is

$$H(s) = H_p(s) \quad \left| \quad s = w_c / s \right. \quad (2.31)$$

Again using Equations 1.6 and 1.11, the first-order transformation matrix is

$$A(1) = \begin{bmatrix} 0 & 1 \\ w_c & 0 \end{bmatrix} \quad (2.32)$$

The method of convolution of coefficients can be used to get the higher-order transformation matrices. However, notice that Equation 2.32 is the same matrix as the lowpass to lowpass transformation with the reciprocal taken of each non zero element, and then each row reversed. This can be shown to be the case in general, so that the lowpass to highpass transformation matrix is given by

$$A(n) = \begin{bmatrix} 0 & 0 & . & . & . & 0 & 1 \\ 0 & 0 & . & . & . & w_c & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & w_c^{n-1} & . & . & . & 0 & 0 \\ w_c^n & 0 & . & . & . & 0 & 0 \end{bmatrix} \quad (2.33)$$

Again, each new filter coefficient depends only on one multiplication although the calculation of the matrix element itself will require calculations involving raising the design constant, w_c , to the $(n-1)^{th}$ power where i is the power of s in the final filter transfer function.

I. LOWPASS TO BANDPASS ANALOG TRANSFORMATION

The polynomial substitution for transforming a lowpass analog prototype to a bandpass analog filter is

$$H(s) = H_p(s) \left| \begin{array}{l} s^2 + w_o^2 \\ s = \frac{Bs}{s^2 + w_o^2} \end{array} \right. \quad (2.34)$$

where w_o is defined as the geometric mean of the passband; $w_o = (w_u w_l)^{1/2}$, and B is the width of the passband, $B = w_u - w_l$. (w_u and w_l are the upper and lower critical frequencies respectively). Again using Equations 1.6 and 1.11 and picking out the appropriate matrix elements, the first-order transformation matrix is given by

$$A(1) = \begin{bmatrix} 0 & B & 0 \\ w_0^2 & 0 & 1 \end{bmatrix} \quad (2.35)$$

The convolution of coefficients technique is again used to generate the higher order transformation matrices with the results for the first-order to third-order matrices listed in Table 4. As with the bandpass digital transformation, the analog bandpass transformation will double the order of the original transfer function since the substitution polynomial is of second order.

TABLE 4
FIRST-ORDER TO THIRD-ORDER BANDPASS
ANALOG TRANSFORMATION MATRICES

$$A(1) = \begin{bmatrix} 0 & B & 0 \\ w_0^2 & 0 & 1 \end{bmatrix}$$

$$A(2) = \begin{bmatrix} 0 & 0 & B^2 & 0 & 0 \\ 0 & Bw_0^2 & 0 & B & 0 \\ w_0^4 & 0 & 2w_0^2 & 0 & 1 \end{bmatrix}$$

$$A(3) = \begin{bmatrix} 0 & 0 & 0 & B^3 & 0 & 0 & 0 \\ 0 & 0 & B^2 w_0^2 & 0 & B^2 & 0 & 0 \\ 0 & Bw_0^4 & 0 & 2Bw_0^2 & 0 & B & 0 \\ w_0^6 & 0 & 3w_0^4 & 0 & 3w_0^2 & 0 & 1 \end{bmatrix}$$

Note that each matrix is slightly over half empty and that the nonzero elements are simple functions of the design constants w_0 and B .

J. LOWPASS TO BANDSTOP ANALOG TRANSFORMATION

For the analog bandstop transformation, the polynomial substitution is the reciprocal of the bandpass case:

$$H(s) = H_b(s) \left| s = \frac{Bs}{s^2 + w_0^2} \right. \quad (2.36)$$

Using the same method as before, the first-order transformation matrix is found to be:

$$A(1) = \begin{bmatrix} w_0^2 & 0 & 1 \\ 0 & B & 0 \end{bmatrix} \quad (2.37)$$

Note that this is the same matrix as the bandpass case but with the rows in reverse order. Using the convolution of coefficients technique the higher-order matrices are developed and the results for the first-order to third-order transformation matrices are shown in Table 5. Notice that the reversal of the order of the rows compared with the bandpass case is true in general. Therefore, once again the matrices are over half empty and nonzero elements are simple functions of B and w_0 .

TABLE 5
FIRST-ORDER TO THIRD-ORDER BANDSTOP
ANALOG TRANSFORMATION MATRICES

$$A(1) = \begin{bmatrix} w_0^2 & 0 & 1 \\ 0 & B & 0 \end{bmatrix}$$

$$A(2) = \begin{bmatrix} w_0^4 & 0 & 2w_0^2 & 0 & 1 \\ 0 & Bw_0^4 & 0 & B & 0 \\ 0 & 0 & B^2 & 0 & 0 \end{bmatrix}$$

$$A(3) = \begin{bmatrix} w_0^6 & 0 & 2w_0^4 & 0 & 3w_0^2 & 0 & 1 \\ 0 & Bw_0^4 & 0 & 2Bw_0^2 & 0 & B & 0 \\ 0 & 0 & B^2 w_0^2 & 0 & B^2 & 0 & 0 \\ 0 & 0 & 0 & B^3 & 0 & 0 & 0 \end{bmatrix}$$

K. GENERAL OBSERVATIONS

Now that the various transformation matrices for digital filter design have been developed, it is possible to compare the two methods of digital filter design mentioned earlier and depicted in Figure 1. For this comparison, consider an analog lowpass prototype filter transfer function with numerator of order m , and denominator of order n (m less than or equal to n).

First consider the digital to digital direct design method. Transforming the analog prototype to a digital prototype using the bilinear transformation will require $m+1$

multiplications and additions per digital prototype coefficient for the numerator polynomial, and $n+1$ multiplications and additions per digital prototype coefficient for the denominator polynomial. Then, each of the digital to digital transformations will require $2(n+1)$ multiplications and additions since a result of the bilinear transformation is to make $n = m$. Therefore, going from an analog prototype to a final digital filter will require $(3n+m+4)$ multiplications and additions. If lowpass digital prototypes are developed prior to the design process, this can be reduced to $2(n+1)$ since then only the digital to digital transformations will be used in designing the final filter.

Now consider the analog design method for the same case of an m^{th} -order numerator and n^{th} -order denominator lowpass prototype transfer function. For the lowpass and highpass case, each new coefficient will require only 1 multiplication and addition. For the bandpass and bandstop transformations a simple expression for the number of calculations per coefficient is difficult to find, however referring to Tables 4 and 5, the number of multiplications and additions per coefficient is always less than half the order of the polynomial to be transformed, on the average. This gives approximately $(n+m)/2$ for the bandpass to bandstop analog transformations. Finally the analog filter must be converted to a digital filter using the bilinear transformation which

will require $(n+m+2)$ calculations per coefficient for the highpass and lowpass case and $2(n+m+1)$ for the bandpass and bandstop case. The total number of calculations in using the analog design method then is $(n+m+4)$ for the high/low pass and $(2.5(n+m)+2)$ for the bandpass/stop. These results are summarized in Table 6.

TABLE 6
NUMBER OF MULTIPLICATIONS AND ADDITIONS REQUIRED
PER COEFFICIENT TO FIND THE DIGITAL FILTER
COEFFICIENTS FROM A LOWPASS ANALOG PROTOTYPE
(WITH AN M^{TH} ORDER NUMERATOR AND N^{TH} ORDER DENOMINATOR)

<u>Analog Design</u>		
	high/low pass	bandpass/stop
analog transformations	2	.5(n+m)
bilinear transformation	n+m+2	2(n+m)+2
Total	n+m+4	2.5(n+m)+2

<u>Digital Design</u>		
	high/low pass	bandpass/stop
bilinear transformation	n+m+2	n+m+2m
digital transformations	2(n+1)	2(n+1)
Total	3(n+1)+m+1	3(n+1)+m+1

Referring to Table 6, the advantage of one method over the other depends upon what type of filter is being designed and the order of the prototype transfer functions numerator and denominator. Consider designing a fifth-order low/highpass Chebyshev or Butterworth filter. In this case, the order of the numerator polynomial is zero and the analog method will have a slight advantage of 9 multiplications and additions versus 12 for the digital design with precalculated prototypes (and 19 for digital design without precalculated prototypes). However, if a bandpass/stop elliptic filter was being designed with $n=5$, then $m=4$ and the direct digital design method (with precalculated prototypes) has an advantage of 12 multiplications and additions compared to approximately 24 for the analog method.

A final consideration in favor of analog design is the complexity of the elements of the transformation matrices. Since the elements of the digital transformation matrices require much more calculation than the elements of the analog transformation matrices or the bilinear transformation (compare Tables 1-5), the problem of roundoff error is much more serious in the case of the digital to digital design method.

L. IMPLEMENTATION

The final step is to implement the algorithms developed previously in a computer program to assist in the teaching of digital filter design. Since the program is intended to be

used for instructional purposes, the numerical considerations discussed earlier are not considered as important as conceptual simplicity. The direct design method was chosen for implementation as the design constants can be found easily without reference to "prewarping." The completed program, called 'DFCADD' (Digital Filter Computer Aided Direct Design), is contained in Appendix A with its own documentation. The program follows the procedures set forth in Chapter 10 of Reference 2. Prototype coefficients are stored within the program for first-order to fifth-order Butterworth and Chebyshev (1/2, 1, & 2 dB ripples) filters. While second-order to sixth-order elliptic lowpass prototype coefficients are calculated within the program for any combination of 1/2, 1, 2, and 3 dB passband ripple and -20, -30, -40, -50, -60, and -70 dB stopband attenuation, with the help of a program developed by Professor D.E. Kirk of the Naval Postgraduate School and References 4 and 5. As a final option, the user may enter his/her own prototype coefficients for up to a tenth-order analog prototype. After calculating the digital filter coefficients an option exists for the program to create a data file, titled "filter", with the digital filter magnitude and phase as a function of the digital frequency from zero to π . This data file can then be used with any plotting routine to generate a plot of the frequency response of the filter.

LIST OF REFERENCES

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2. Strum, R.D. and Kirk, D.E., First Principles of Discrete Systems and Digital Signal Processing, pp. 612-701, Addison-Wesley, 1988.
3. Constantinides, A.G., "Spectral Transformations for Digital Filters, "Proceedings of IEEE, p. 117,(August): 1585-1590.
4. Ludeman, L.C., Fundamentals of Digital Signal Processing, pp. 149-160, Harper and Row, 1986.
5. Antoniou, A., Digital Filters: Analysis and Design, pp. 114-125, McGraw-Hill, 1979.

APPENDIX DFCADD FORTRAN PROGRAM

FILE: DFCADD FORTRAN A

```

C *****
C * THIS PROGRAM ASSISTS IN THE CALCULATION OF DIGITAL FILTER * NEW00010
C * COEFFICIENTS ACCORDING TO THE PROCEDURE SET FORTH IN CHAPTER * NEW00020
C * 10 (SUMMARIZED ON PG 695) OF - FIRST PRINCIPLES OF DISCRETE * NEW00030
C * SYSTEMS AND DIGITAL SIGNAL PROCESSING - BY R.D. STRUM AND * NEW00040
C * D.E. KIRK (REFERENCE(2)) * NEW00050
C ***** NEW00060
C ***** NEW00070
C INTEGER ORDER,TYPE,PASS,ORDER2,END,AORDF NEW00080
C REAL KPR,LAM,OMEGA(20),V(20),AO(20),BO(20),B1(20) NEW00090
C COMPLEX Z,NEWZ,ZN,ZD,HOFZN,HOFZD,HOFZ NEW00100
C DOUBLE PRECISION TEMP,SUMR NEW00110
C DOUBLE PRECISION R,S0,A01,A02,B01,B02,B11,B12,H0 NEW00120
C DOUBLE PRECISION A03,A04,B03,B04,B13,B14 NEW00130
C DOUBLE PRECISION SFREQ,AFC,DFC,ALPHA,PI,PID4,DFCD2 NEW00140
C DOUBLE PRECISION ALC,AUC,DLCD,DUC,RK,B,C,DLCD2,DUCD2 NEW00150
C DOUBLE PRECISION APROT,BILINR,DPROT,LPHPTR,BPBSTR,DFLTR NEW00160
C ***** NEW00170
C FORMAT OF ARRAYS: NEW00180
C -APROT CONTAINS THE ANALOG PROTOTYPE NEW00190
C APROT(NORD,PWR) NEW00200
C -BILINR IS THE ANALOG-DIGITAL BILINEAR TRANSFORMATION MATRIX NEW00210
C BILINR(ORDER,ROW,COL) NEW00220
C -DPROT IS THE DIGITAL PROTOTYPE FROM APROT AND BILINR NEW00230
C DPROT(NORD,PWR) NEW00240
C -LPHPTR IS THE LP&HP DIGITAL-DIGITAL TRANSFORMATION MATRIX NEW00250
C LPHPTR(ORDER,ROW,COL) NEW00260
C -DFLTR IS THE FINAL DIGITAL FILTER NEW00270
C DFLTR(NORD,PWR) NEW00280
C -BPBSTR IS THE DP&BS DIGITAL-DIGITAL TRANSFORMATION MATRIX NEW00290
C BPBSTR(ORDER,ROW,COL) NEW00300
C ***** NEW00310
C DIMENSION APROT(2,0:10) NEW00320
C DIMENSION BILINR(10,0:10,0:10) NEW00330
C DIMENSION DPROT(2,0:10) NEW00340
C DIMENSION LPHPTR(10,0:10,0:10) NEW00350
C DIMENSION BPBSTR(10,0:10,0:20) NEW00360
C DIMENSION DFLTR(2,0:20) NEW00370
C ***** NEW00380
C OPEN(UNIT=8,FILE='FILTER DATA',STATUS='UNKNOWN') NEW00390
C ***** NEW00400
C PI=3.1415926535898D+00 NEW00410
C SPI=3.14159 NEW00420
C PID4=PI/4.0D+00 NEW00430
C ***** NEW00440
C CLEAR APROT PRIOR TO LOADING NEW00450
C DO 1 K=1,2 NEW00460
C DO 2 L=0,10 NEW00470
C APROT(K,L)=0.0D+00 NEW00480
C CONTINUE NEW00490
C CONTINUE NEW00500
C ***** NEW00510
C ***** NEW00520
C BEGIN INTERACTION NEW00530
C CALL EXCMS ('CLRSCRN') NEW00540
C WRITE(6,*) 'THIS IS A PROGRAM TO CALCULATE DIGITAL FILTERS USING' NEW00550
C WRITE(6,*) ' THE DIRECT DESIGN METHOD.' NEW00560
C WRITE(6,*) NEW00570
C WRITE(6,*) '*****' NEW00580
C WRITE(6,*) '*WARNING-THIS PROGRAM IS NOT USER FRIENDLY!!! *' NEW00590
C WRITE(6,*) '* YOU MUST ENTER VALUES OF THE APPROPRIATE TYPE*' NEW00600
C WRITE(6,*) '* (REAL OR INTEGER), AND PROPER RANGE *' NEW00610
C WRITE(6,*) '*****' NEW00620
C WRITE(6,*) NEW00630
C WRITE(6,*) ' ** ENTER 1 TO CONTINUE ** ' NEW00640
C READ(5,*) LOOK NEW00650
C CALL EXCMS ('CLRSCRN') NEW00660
C 9 WRITE(6,*) 'WHAT TYPE OF FILTER DO YOU WANT?' NEW00670
C 10 WRITE(6,*) ' (1-4 ALLOW ONLY UP TO 5TH ORDER PROTOTYPES)' NEW00680
C WRITE(6,*) ' 1-BUTTERWORTH ' NEW00690
C WRITE(6,*) ' 2-CHEBYSHEV WITH 1/2 DB RIPPLE ' NEW00700
C WRITE(6,*) ' 3-CHEBYSHEV WITH 1 DB RIPPLE ' NEW00710
C WRITE(6,*) ' 4-CHEBYSHEV WITH 2 DB RIPPLE' NEW00720

```

FILE: DFCADD FORTRAN A

```

WRITE(6,*) ' 5-ELLIPTIC (ALLOWS UP TO A 6TH ORDER PROTOTYPE)' NEW00730
WRITE(6,*) ' -PASSBAND RIPPLE 0.5,1.0,2.0,OR 3.0 DB' NEW00740
WRITE(6,*) ' -STOPBAND ATTENUATION -20,-30,-40,-50,-60, ' NEW00750
WRITE(6,*) ' OR -70 DB ' NEW00760
WRITE(6,*) ' 6-OTHER - ALLOWS YOU TO USE UP TO A 10TH ORDER' NEW00770
WRITE(6,*) ' PROTOTYPE, HOWEVER YOU MUST ENTER THE ANALOG' NEW00780
WRITE(6,*) ' PROTOTYPE COEFFICIENTS ' NEW00790
WRITE(6,*) ' ** INTEGER 1-6 **' NEW00800
READ(5,*) TYPE NEW00810
***** NEW00820
C CALL EXCMS ('CLRSCRN') NEW00830
14 WRITE(6,*) NEW00840
15 WRITE(6,*) 'WHAT IS THE ORDER OF THE PROTOTYPE YOU WISH TO USE?' NEW00850
WRITE(6,*) ' - MUST BE AN INTEGER IN THE RANGE 1-5 UNLESS YOU' NEW00860
WRITE(6,*) ' CHOSE TO ENTER YOUR OWN PROTOTYPE (THEN 1-10)' NEW00870
WRITE(6,*) ' OR YOU CHOSE AN ELLIPTIC FILTER (THEN 1-6)' NEW00880
WRITE(6,*) ' - OR ENTER "0" IF YOU WANT HELP SELECTING THE ORDER' NEW00890
WRITE(6,*) ' BY CHANGING SPECIFICATION FREQUENCIES TO THE' NEW00900
WRITE(6,*) ' CORRESPONDING DIGITAL PROTOTYPE FREQUENCIES ' NEW00910
WRITE(6,*) ' NOTE: YOU WILL NEED PROTOTYPE FREQUENCY ' NEW00920
WRITE(6,*) ' RESPONSE PLOTS LIKE THOSE GIVEN IN REF(2) FOR ' NEW00930
WRITE(6,*) ' BUTTERWORTH FILTERS (FIG 10.33, PGS 691&692) ' NEW00940
WRITE(6,*) ' AND CHEBYSHEV FILTERS (FIG 10.34, PGS 696-698) ' NEW00950
READ(5,*) ORDER NEW00960
IF((TYPE.GT.4).AND.(TYPE.LT.8))THEN NEW00970
  IF(ORDER.EQ.1)THEN NEW00980
    WRITE(6,*) 'ELLIPTIC FILTERS GIVEN FOR 2ND-9TH ORDER ONLY' NEW00990
    WRITE(6,*) NEW01000
    GO TO 10 NEW01010
  ENDIF NEW01020
ENDIF NEW01030
C CALL EXCMS ('CLRSCRN') NEW01040
WRITE(6,*) 'DO YOU WANT TO ENTER SPECIFICATION FREQUENCIES' NEW01050
WRITE(6,*) 'AS ANALOG OR DIGITAL FREQUENCIES?' NEW01060
WRITE(6,*) ' 1-ANALOG ' NEW01070
WRITE(6,*) ' 2-DIGITAL ' NEW01080
WRITE(6,*) ' ** INTEGER 1 OR 2 ** ' NEW01090
READ(5,*) AFORDF NEW01100
CALL EXCMS ('CLRSCRN') NEW01110
IF(AFORDF.EQ.1)THEN NEW01120
  WRITE(6,*) 'WHAT IS THE SAMPLING FREQUENCY IN KHZ?' NEW01130
  WRITE(6,*) ' ** DECIMAL **' NEW01140
  READ(5,*) SSFREQ NEW01150
  SFREQ=DBLE(SSFREQ) NEW01160
ENDIF NEW01170
CALL EXCMS ('CLRSCRN') NEW01180
WRITE(6,*) 'WHAT TYPE OF PASSBAND DUE YOU WANT?' NEW01190
WRITE(6,*) ' 1-LOWPASS' NEW01200
WRITE(6,*) ' 2-HIGHPASS' NEW01210
WRITE(6,*) ' 3-BANDPASS' NEW01220
WRITE(6,*) ' 4-BANDSTOP' NEW01230
WRITE(6,*) ' ** INTEGER 1-4 ** ' NEW01240
READ(5,*) PASS NEW01250
C FORMULAS FOR HIGHPASS AND LOWPASS NEW01260
IF((PASS.EQ.1).OR.(PASS.EQ.2))THEN NEW01270
  CALL EXCMS ('CLRSCRN') NEW01280
  IF(AFORDF.EQ.1)THEN NEW01290
    WRITE(6,*) 'WHAT IS THE CRITICAL FREQUENCY IN KHZ? ' NEW01300
    WRITE(6,*) ' - HEG 3DB PT FOR BUTTERWORTH ' NEW01310
    WRITE(6,*) ' - RIPPLE EDGE FOR ELLIPTIC AND CHEBYSHEV' NEW01320
    WRITE(6,*) ' ** DECIMAL ** ' NEW01330
    READ(5,*) SAFC NEW01340
    AFC=DBLE(SAFC) NEW01350
    DFC=(AFC/SFREQ)*PI*(2.0D+00) NEW01360
  ENDIF NEW01370
  IF(AFORDF.EQ.2)THEN NEW01380
    WRITE(6,*) 'WHAT IS THE CRITICAL DIGITAL FREQUENCY?' NEW01390
  ENDIF NEW01400
  NEW01410
  NEW01420
  NEW01430
  NEW01440

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WRITE(6,*) ' - NEG 3DB FOR BUTTERWORTH ' NEW01450
WRITE(6,*) ' - RIPPLE EDGE FOR ELLIPTIC AND CHEBYSHEV' NEW01460
WRITE(6,*) ' ** DECIMAL ** ' NEW01470
READ(5,*) SDFC NEW01480
DFC=DBLE(SDFC) NEW01490
ENDIF NEW01500
DFCD2=DFC/2.0D+00 NEW01510
C CALCULATE THE APPROPRIATE VALUES OF THE DESIGN CONSTANT ALPHA NEW01520
C FROM TABLE 10.7 REFERENCE (2) AND REFERENCE (3) NEW01530
IF(PASS.EQ.1)THEN NEW01540
  ALPHA=DSIN((PID4)-(DFCD2))/DSIN((PID4)+(DFCD2)) NEW01550
ENDIF NEW01560
IF(PASS.EQ.2)THEN NEW01570
  ALPHA=DCOS((PID4)-(DFCD2))/DCOS((PID4)+(DFCD2)) NEW01580
ENDIF NEW01590
C ***** NEW01600
C OPTION TO SHOW CALCULATED VALUE OF ALPHA NEW01610
CALL EXCMS ('CLRSCRN') NEW01620
WRITE(6,*) 'DO YOU WISH TO SEE THE CALCULATED VALUE OF ALPHA?' NEW01630
WRITE(6,*) ' ** 1-YES/2-NO **' NEW01640
READ(5,*) LOOK NEW01650
IF(LOOK.EQ.1)THEN NEW01660
  WRITE(6,*) NEW01670
  WRITE(6,903) ALPHA NEW01680
  WRITE(6,*) NEW01690
  WRITE(6,*) '** ENTER 1 TO CONTINUE **' NEW01700
  READ(5,*) LOOK NEW01710
ENDIF NEW01720
C ***** NEW01730
C BANDPASS AND BANDSTOP FORMULAS NEW01740
IF((PASS.EQ.3).OR.(PASS.EQ.4))THEN NEW01750
  CALL EXCMS ('CLRSCRN') NEW01760
  IF(AFORDF.EQ.1)THEN NEW01770
    WRITE(6,*) 'WHAT IS THE LOWER CRITICAL FREQUENCY IN KHZ?' NEW01780
    WRITE(6,*) ' - NEG 3DB FOR BUTTERWORTH' NEW01790
    WRITE(6,*) ' - RIPPLE EDGE FOR ELLIPTIC AND CHEBYSHEV ' NEW01800
    WRITE(6,*) ' ** DECIMAL **' NEW01810
    READ(5,*) SALC NEW01820
    ALC=DBLE(SALC) NEW01830
    CALL EXCMS ('CLRSCRN') NEW01840
    WRITE(6,*) 'WHAT IS THE UPPER CRITICAL FREQUENCY IN KHZ?' NEW01850
    WRITE(6,*) ' - NEG 3DB FOR BUTTERWORTH' NEW01860
    WRITE(6,*) ' - RIPPLE EDGE FOR ELLIPTIC AND CHEBYSHEV ' NEW01870
    WRITE(6,*) ' ** DECIMAL **' NEW01880
    READ(5,*) SAUC NEW01890
    AUC=DBLE(SAUC) NEW01900
    DLC=(ALC/SFREQ)*(2.0D+00)*PI NEW01910
    DUC=(AUC/SFREQ)*(2.0D+00)*PI NEW01920
  ENDIF NEW01930
  IF(AFORDF.EQ.2)THEN NEW01940
    WRITE(6,*) 'WHAT IS THE LOWER CRITICAL DIGITAL FREQUENCY?' NEW01950
    WRITE(6,*) ' - NEG 3DB FOR BUTTERWORTH ' NEW01960
    WRITE(6,*) ' - RIPPLE EDGE FOR CHEBYSHEV AND ELLIPTIC' NEW01970
    WRITE(6,*) ' ** DECIMAL ** ' NEW01980
    READ(5,*) SDLC NEW01990
    DLC=DBLE(SDLC) NEW02000
    CALL EXCMS ('CLRSCRN') NEW02010
    WRITE(6,*) 'WHAT IS THE UPPER CRITICAL DIGITAL FREQUENCY?' NEW02020
    WRITE(6,*) ' - NEG 3DB FOR BUTTERWORTH ' NEW02030
    WRITE(6,*) ' - RIPPLE EDGE FOR CHEBYSHEV AND ELLIPTIC' NEW02040
    WRITE(6,*) ' ** DECIMAL ** ' NEW02050
    READ(5,*) SDUC NEW02060
    DUC=DBLE(SDUC) NEW02070
  ENDIF NEW02080
  DLCD2=DLC/2.0D+00 NEW02090
  DUCD2=DUK/2.0D+00 NEW02100
C CALCULATE APPROPRIATE VALUES OF DESIGN CONSTANTS ALPHA AND K NEW02110
C FROM TABLE 10.7 REFERENCE (2) AND REFERENCE (3) NEW02120
ALPHA=DCOS((DUCD2)+(DLCD2))/DCOS((DUCD2)-(DLCD2)) NEW02130
IF(PASS.EQ.4)THEN NEW02140
  RK=DTAN(PID4)*DTAN((DUCD2)-(DLCD2)) NEW02150
ENDIF NEW02160

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      B=((2.0D+00)*ALPHA)/(RK+1.0D+00)
      C=((1.0D+00)-RK)/((1.0D+00)+RK)
    ENDIF
    IF(PASS.EQ.3)THEN
      RK=DTAN(PID4)/DTAN((DUCD2)-(DLCD2))
      B=((2.0D+00)*ALPHA*RK)/(RK+(1.0D+00))
      C=(RK-(1.0D+00))/(RK+(1.0D+00))
    ENDIF
C    OPTION TO SHOW CALCULATED VALUE OF ALPHA AND K
      CALL EXCMS ('CLRSCRN')
      WRITE(6,*) 'DO YOU WISH TO SEE THE CALCULATED VALUE OF ALPHA '
      WRITE(6,*) ' AND K? '
      WRITE(6,*) ' ** 1-YES/2-NO **'
      READ(5,*) LOOK
      IF(LOOK.EQ.1)THEN
        WRITE(6,*)
        WRITE(6,903) ALPHA
        WRITE(6,904) RK
        WRITE(6,*)
        WRITE(6,*) '** ENTER 1 TO CONTINUE **'
        READ(5,*) LOOK
      ENDIF
C    *****
C    OPTION TO ASSIST IN DETERMINING THE PROTOTYPE ORDER BY CHANGING
C    SPECIFICATION FREQUENCIES TO APPROPRIATE PROTOTYPE FREQUENCIES
C    FROM EQN 10.274 REFERENCE(1)
      IF(ORDER.EQ.0)THEN
16      CALL EXCMS ('CLRSCRN')
        IF(AFORDF.EQ.1)THEN
          WRITE(6,*) 'WHAT IS THE ANALOG SPECIFICATION FREQUENCY (KHZ)'
          WRITE(6,*) 'THAT YOU WANT TO CONVERT TO A DIGITAL'
          WRITE(6,*) 'LOWPASS PROTOTYPE FREQUENCY?'
          WRITE(6,*) ' ** DECIMAL **'
          READ(5,*) F
          DF=(2.0)*SPI*(F/SSFREQ)
        ENDIF
        IF(AFORDF.EQ.2)THEN
          WRITE(6,*) 'WHAT IS THE DIGITAL SPECIFICATION FREQUENCY'
          WRITE(6,*) 'THAT YOU WANT TO CONVERT TO A DIGITAL'
          WRITE(6,*) 'LOWPASS PROTOTYPE FREQUENCY?'
          WRITE(6,*) ' ** DECIMAL **'
          READ(5,*) DF
        ENDIF
        X=COS(DF)
        Y=SIN(DF)
        Z=CMPLX(X,Y)
        IF(PASS.EQ.1)THEN
          AS=REAL(ALPHA)
          NEWZ=(Z-AS)/((1.0)-(AS*Z))
        ENDIF
        IF(PASS.EQ.2)THEN
          AS=REAL(ALPHA)
          NEWZ=(-1.0)*((Z-AS)/((1.0)-(AS*Z)))
        ENDIF
        IF(PASS.EQ.3)THEN
          BS=REAL(B)
          CS=REAL(C)
          NEWZ=(-1.0)*(((Z**2)-(BS*Z)+CS)/((1.0)-(BS*Z)+(CS*(Z**2))))
        ENDIF
        IF(PASS.EQ.4)THEN
          BS=REAL(B)
          CS=REAL(C)
          NEWZ=((Z**2)-(BS*Z)+CS)/((1.0)-(BS*Z)+(CS*(Z**2)))
        ENDIF
        PTHETA=ABS(ATAN(AIMAG(NEWZ)/REAL(NEWZ)))
        IF(PASS.EQ.1)THEN
          IF(F.GT.SAFC)THEN
            PTHETA=SPI-PTHETA
          ENDIF
        ENDIF
        IF(PASS.EQ.2)THEN

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      IF(F.LT.SAFC)THEN
        PTHETA=SPI-PTHETA
      ENDIF
    ENDIF
    IF(PASS.EQ.3)THEN
      IF((F.LT.SALC).OR.(F.GT.SAUC))THEN
        PTHETA=SPI-PTHETA
      ENDIF
    ENDIF
    IF(PASS.EQ.4)THEN
      IF((F.GT.SALC).AND.(F.LT.SAUC))THEN
        PTHETA=SPI-PTHETA
      ENDIF
    ENDIF
    WRITE(6,905) PTHETA
    WRITE(6,*)
    WRITE(6,*) 'WANT TO CONVERT ANOTHER FREQUENCY?'
    WRITE(6,*) ' ** 1-YES/2-NO **'
    READ(5,*) LOOK
    IF(LOOK.EQ.1)GO TO 16
    IF(LOOK.EQ.2)THEN
      WRITE(6,*)
      WRITE(6,*) 'WHAT ORDER PROTOTYPE DO YOU WANT TO USE?'
      WRITE(6,*) ' ** INTEGER 1-5 ** '
      READ(5,*) ORDER
    ENDIF
  ENDIF
C *****
C LOAD APROT MATRIX WITH APPROPRIATE COEFFICIENTS
C APROT(TYPE=1) ARE BUTTERWORTH FILTERS
  IF(TYPE.EQ.1)THEN
    IF(ORDER.EQ.1)THEN
      APROT(1,0)=1.0D+00
      APROT(2,0)=1.0D+00
      APROT(2,1)=1.0D+00
    ELSEIF(ORDER.EQ.2)THEN
      APROT(1,0)=1.0D+00
      APROT(2,0)=1.0D+00
      APROT(2,1)=DSQRT(2.0D+00)
      APROT(2,2)=1.0D+00
    ELSEIF(ORDER.EQ.3)THEN
      APROT(1,0)=1.0D+00
      APROT(2,0)=1.0D+00
      APROT(2,1)=2.0D+00
      APROT(2,2)=2.0D+00
      APROT(2,3)=1.0D+00
    ELSEIF(ORDER.EQ.4)THEN
      APROT(1,0)=1.0D+00
      APROT(2,0)=1.0D+00
      APROT(2,1)=2.613126D+00
      APROT(2,2)=3.414214D+00
      APROT(2,3)=2.613126D+00
      APROT(2,4)=1.0D+00
    ELSEIF(ORDER.EQ.5)THEN
      APROT(1,0)=1.0D+00
      APROT(2,0)=1.0D+00
      APROT(2,1)=3.236068D+00
      APROT(2,2)=5.236068D+00
      APROT(2,3)=5.236068D+00
      APROT(2,4)=3.236068D+00
      APROT(2,5)=1.0D+00
    ENDIF
  ENDIF
C APROT(TYPE=2) ARE CHEBYSHEV FILTERS WITH .5DB RIPPLE
  IF(TYPE.EQ.2)THEN
    IF(ORDER.EQ.1)THEN
      APROT(1,0)=2.862775D+00
      APROT(2,0)=2.862775D+00
      APROT(2,1)=1.0D+00
    ELSEIF(ORDER.EQ.2)THEN
      APROT(1,0)=1.431388D+00
      APROT(2,0)=1.516203D+00

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NEW02890
NEW02900
NEW02910
NEW02920
NEW02930
NEW02940
NEW02950
NEW02960
NEW02970
NEW02980
NEW02990
NEW03000
NEW03010
NEW03020
NEW03030
NEW03040
NEW03050
NEW03060
NEW03070
NEW03080
NEW03090
NEW03100
NEW03110
NEW03120
NEW03130
NEW03140
NEW03150
NEW03160
NEW03170
NEW03180
NEW03190
NEW03200
NEW03210
NEW03220
NEW03230
NEW03240
NEW03250
NEW03260
NEW03270
NEW03280
NEW03290
NEW03300
NEW03310
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NEW03340
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NEW03390
NEW03400
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NEW03500
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NEW03520
NEW03530
NEW03540
NEW03550
NEW03560
NEW03570
NEW03580
NEW03590
NEW03600

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APROT(2,1)=1.425625D+00	NEW03610
APROT(2,2)=1.0D+00	NEW03620
ELSEIF(ORDER.EQ.3)THEN	NEW03630
APROT(1,0)=0.715694D+00	NEW03640
APROT(2,0)=0.715694D+00	NEW03650
APROT(2,1)=1.534895D+00	NEW03660
APROT(2,2)=1.252913D+00	NEW03670
APROT(2,3)=1.0D+00	NEW03680
ELSEIF(ORDER.EQ.4)THEN	NEW03690
APROT(1,0)=0.357847D+00	NEW03700
APROT(2,0)=0.379051D+00	NEW03710
APROT(2,1)=1.025455D+00	NEW03720
APROT(2,2)=1.716866D+00	NEW03730
APROT(2,3)=1.197386D+00	NEW03740
APROT(2,4)=1.0D+00	NEW03750
ELSEIF(ORDER.EQ.5)THEN	NEW03760
APROT(1,0)=0.178923D+00	NEW03770
APROT(2,0)=0.178923D+00	NEW03780
APROT(2,1)=0.752518D+00	NEW03790
APROT(2,2)=1.309575D+00	NEW03800
APROT(2,3)=1.937368D+00	NEW03810
APROT(2,4)=1.172491D+00	NEW03820
APROT(2,5)=1.0D+00	NEW03830
ENDIF	NEW03840
ENDIF	NEW03850
C APROT(TYPE=3) ARE CHEBYSHEV FILTERS WITH 1DB RIPPLE	NEW03860
IF(TYPE.EQ.3)THEN	NEW03870
IF(ORDER.EQ.1)THEN	NEW03880
APROT(1,0)=1.196523D+00	NEW03890
APROT(2,0)=1.196523D+00	NEW03900
APROT(2,1)=1.0D+00	NEW03910
ELSEIF(ORDER.EQ.2)THEN	NEW03920
APROT(1,0)=0.982613D+00	NEW03930
APROT(2,0)=1.102510D+00	NEW03940
APROT(2,1)=1.097734D+00	NEW03950
APROT(2,2)=1.0D+00	NEW03960
ELSEIF(ORDER.EQ.3)THEN	NEW03970
APROT(1,0)=0.491307D+00	NEW03980
APROT(2,0)=0.491307D+00	NEW03990
APROT(2,1)=1.238409D+00	NEW04000
APROT(2,2)=0.988341D+00	NEW04010
APROT(2,3)=1.0D+00	NEW04020
ELSEIF(ORDER.EQ.4)THEN	NEW04030
APROT(1,0)=0.245653D+00	NEW04040
APROT(2,0)=0.275628D+00	NEW04050
APROT(2,1)=0.742619D+00	NEW04060
APROT(2,2)=1.453925D+00	NEW04070
APROT(2,3)=0.952811D+00	NEW04080
APROT(2,4)=1.0D+00	NEW04090
ELSEIF(ORDER.EQ.5)THEN	NEW04100
APROT(1,0)=0.122827D+00	NEW04110
APROT(2,0)=0.122827D+00	NEW04120
APROT(2,1)=0.580534D+00	NEW04130
APROT(2,2)=0.974396D+00	NEW04140
APROT(2,3)=1.688816D+00	NEW04150
APROT(2,4)=0.936820D+00	NEW04160
APROT(2,5)=1.0D+00	NEW04170
ENDIF	NEW04180
ENDIF	NEW04190
C APROT(TYPE=4) ARE CHEBYSHEV FILTERS WITH 2DB RIPPLE	NEW04200
IF(TYPE.EQ.4)THEN	NEW04210
IF(ORDER.EQ.1)THEN	NEW04220
APROT(1,0)=1.307560D+00	NEW04230
APROT(2,0)=1.307560D+00	NEW04240
APROT(2,1)=1.0D+00	NEW04250
ELSEIF(ORDER.EQ.2)THEN	NEW04260
APROT(1,0)=0.505803D+00	NEW04270
APROT(2,0)=0.636768D+00	NEW04280
APROT(2,1)=0.803816D+00	NEW04290
APROT(2,2)=1.0D+00	NEW04300
ELSEIF(ORDER.EQ.3)THEN	NEW04310
APROT(1,0)=0.326890D+00	NEW04320


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      APROT(2,0)=0.326890D+00
      APROT(2,1)=1.022190D+00
      APROT(2,2)=0.737822D+00
      APROT(2,3)=1.0D+00
      ELSEIF(ORDER.EQ.4)THEN
      APROT(1,0)=0.163445D+00
      APROT(2,0)=0.205765D+00
      APROT(2,1)=0.516798D+00
      APROT(2,2)=1.256482D+00
      APROT(2,3)=0.716215D+00
      APROT(2,4)=1.0D+00
      ELSEIF(ORDER.EQ.5)THEN
      APROT(1,0)=0.081723D+00
      APROT(2,0)=0.081723D+00
      APROT(2,1)=0.459349D+00
      APROT(2,2)=0.693477D+00
      APROT(2,3)=1.499543D+00
      APROT(2,4)=0.706461D+00
      APROT(2,5)=1.0D+00
      ENDIF
      ENDIF
C      APROT(TYPE=5) ARE ELLIPTIC FILTERS
C      ELLIPTIC COEFFICIENTS ARE CALCULATED USING A PROGRAM DEVELOPED
C      BY PROFESSOR D. E. KIRK, NAVAL POSTGRADUATE SCHOOL, MONTEREY,
C      CALIFORNIA 93943
      A02=0.0D+00
      B02=0.0D+00
      B12=0.0D+00
      S0=0.0D+00
      IF(TYPE.EQ.5)THEN
      CALL EXCMS ('CLRSCRN')
      WRITE(6,*) 'WHAT PASSBAND RIPPLE DO YOU WANT? (IN DB)'
      WRITE(6,*) '*** MUST BE EITHER 0.5, 1.0, 2.0, OR 3.0 ***'
      READ(5,*) ASUBP
      WRITE(6,*)
      WRITE(6,*)
      WRITE(6,*) 'WHAT STOPBAND ATTENUATION DO YOU WANT? (IN DB)'
      WRITE(6,*) '*** MUST BE -20,-30,-40,-50,-60, OR -70 ***'
      WRITE(6,*) '*** DO NOT INCLUDE A DECIMAL ***'
      READ(5,*) NATTN
      NATTN=-1*NATTN
      ASUBA=REAL(NATTN)
      IF(ASUBP.EQ.0.5)THEN
      IF(NATTN.EQ.20)THEN
      IF(ORDER.EQ.2)SR=2.76261
      IF(ORDER.EQ.3)SR=1.42189
      IF(ORDER.EQ.4)SR=1.13188
      IF(ORDER.EQ.5)SR=1.04465
      IF(ORDER.EQ.6)SR=1.01553
      ENDIF
      IF(NATTN.EQ.30)THEN
      IF(ORDER.EQ.2)SR=4.80880
      IF(ORDER.EQ.3)SR=1.92322
      IF(ORDER.EQ.4)SR=1.32446
      IF(ORDER.EQ.5)SR=1.12912
      IF(ORDER.EQ.6)SR=1.05394
      ENDIF
      IF(NATTN.EQ.40)THEN
      IF(ORDER.EQ.2)SR=8.848925
      IF(ORDER.EQ.3)SR=2.71147
      IF(ORDER.EQ.4)SR=1.62842
      IF(ORDER.EQ.5)SR=1.27264
      IF(ORDER.EQ.6)SR=1.12697
      ENDIF
      IF(NATTN.EQ.50)THEN
      IF(ORDER.EQ.2)SR=15.06069
      IF(ORDER.EQ.3)SR=3.90430
      IF(ORDER.EQ.4)SR=2.06924
      IF(ORDER.EQ.5)SR=1.48469
      IF(ORDER.EQ.6)SR=1.24101
      ENDIF
      IF(NATTN.EQ.60)THEN

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NEW04330
 NEW04340
 NEW04350
 NEW04360
 NEW04370
 NEW04380
 NEW04390
 NEW04400
 NEW04410
 NEW04420
 NEW04430
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 NEW04460
 NEW04470
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 NEW04490
 NEW04500
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 NEW04570
 NEW04580
 NEW04590
 NEW04600
 NEW04610
 NEW04620
 NEW04630
 NEW04640
 NEW04650
 NEW04660
 NEW04670
 NEW04680
 NEW04690
 NEW04700
 NEW04710
 NEW04720
 NEW04730
 NEW04740
 NEW04750
 NEW04760
 NEW04770
 NEW04780
 NEW04790
 NEW04800
 NEW04810
 NEW04820
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 NEW04880
 NEW04890
 NEW04900
 NEW04910
 NEW04920
 NEW04930
 NEW04940
 NEW04950
 NEW04960
 NEW04970
 NEW04980
 NEW04990
 NEW05000
 NEW05010
 NEW05020
 NEW05030
 NEW05040

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      IF (ORDER.EQ.2)SR=6.0503E
      IF (ORDER.EQ.3)SR=6.0503E
      IF (ORDER.EQ.4)SR=6.0503E
      IF (ORDER.EQ.5)SR=6.0503E
      IF (ORDER.EQ.6)SR=6.0503E
    ENDDO
    IF (NATTN.EQ.7) THEN
      IF (ORDER.EQ.2)SR=4.5885E
      IF (ORDER.EQ.3)SR=4.5885E
      IF (ORDER.EQ.4)SR=4.5885E
      IF (ORDER.EQ.5)SR=4.5885E
      IF (ORDER.EQ.6)SR=4.5885E
    ENDDO
    ENDDO
    IF (ASR.EQ.3) THEN
      IF (NATTN.EQ.2) THEN
        IF (ORDER.EQ.2)SR=2.3247E
        IF (ORDER.EQ.3)SR=2.3247E
        IF (ORDER.EQ.4)SR=2.3247E
        IF (ORDER.EQ.5)SR=2.3247E
        IF (ORDER.EQ.6)SR=2.3247E
      ENDDO
      IF (NATTN.EQ.3) THEN
        IF (ORDER.EQ.2)SR=4.1840E
        IF (ORDER.EQ.3)SR=4.1840E
        IF (ORDER.EQ.4)SR=4.1840E
        IF (ORDER.EQ.5)SR=4.1840E
        IF (ORDER.EQ.6)SR=4.1840E
      ENDDO
      IF (NATTN.EQ.4) THEN
        IF (ORDER.EQ.2)SR=7.1448E
        IF (ORDER.EQ.3)SR=7.1448E
        IF (ORDER.EQ.4)SR=7.1448E
        IF (ORDER.EQ.5)SR=7.1448E
        IF (ORDER.EQ.6)SR=7.1448E
      ENDDO
      IF (NATTN.EQ.5) THEN
        IF (ORDER.EQ.2)SR=12.4847E
        IF (ORDER.EQ.3)SR=12.4847E
        IF (ORDER.EQ.4)SR=12.4847E
        IF (ORDER.EQ.5)SR=12.4847E
        IF (ORDER.EQ.6)SR=12.4847E
      ENDDO
      IF (NATTN.EQ.6) THEN
        IF (ORDER.EQ.2)SR=22.1788E
        IF (ORDER.EQ.3)SR=22.1788E
        IF (ORDER.EQ.4)SR=22.1788E
        IF (ORDER.EQ.5)SR=22.1788E
        IF (ORDER.EQ.6)SR=22.1788E
      ENDDO
      IF (NATTN.EQ.7) THEN
        IF (ORDER.EQ.2)SR=39.4028E
        IF (ORDER.EQ.3)SR=39.4028E
        IF (ORDER.EQ.4)SR=39.4028E
        IF (ORDER.EQ.5)SR=39.4028E
        IF (ORDER.EQ.6)SR=39.4028E
      ENDDO
    ENDDO
    IF (ASR.EQ.2) THEN
      IF (NATTN.EQ.2) THEN
        IF (ORDER.EQ.2)SR=1.3400E
        IF (ORDER.EQ.3)SR=1.3400E
        IF (ORDER.EQ.4)SR=1.3400E
        IF (ORDER.EQ.5)SR=1.3400E
        IF (ORDER.EQ.6)SR=1.3400E
      ENDDO
      IF (NATTN.EQ.3) THEN
        IF (ORDER.EQ.2)SR=3.2820E
        IF (ORDER.EQ.3)SR=3.2820E
        IF (ORDER.EQ.4)SR=3.2820E
        IF (ORDER.EQ.5)SR=3.2820E
        IF (ORDER.EQ.6)SR=3.2820E
      ENDDO
    ENDDO
  ENDDO
  IF (ASR.EQ.1) THEN
    IF (NATTN.EQ.2) THEN
      IF (ORDER.EQ.2)SR=0.2820E
      IF (ORDER.EQ.3)SR=0.2820E
      IF (ORDER.EQ.4)SR=0.2820E
      IF (ORDER.EQ.5)SR=0.2820E
      IF (ORDER.EQ.6)SR=0.2820E
    ENDDO
    IF (NATTN.EQ.3) THEN
      IF (ORDER.EQ.2)SR=0.5640E
      IF (ORDER.EQ.3)SR=0.5640E
      IF (ORDER.EQ.4)SR=0.5640E
      IF (ORDER.EQ.5)SR=0.5640E
      IF (ORDER.EQ.6)SR=0.5640E
    ENDDO
  ENDDO

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      NEMO5150
      NEMO5160
      NEMO5170
      NEMO5180
      NEMO5190
      NEMO5200
      NEMO5210
      NEMO5220
      NEMO5230
      NEMO5240
      NEMO5250
      NEMO5260
      NEMO5270
      NEMO5280
      NEMO5290
      NEMO5300
      NEMO5310
      NEMO5320
      NEMO5330
      NEMO5340
      NEMO5350
      NEMO5360
      NEMO5370
      NEMO5380
      NEMO5390
      NEMO5400
      NEMO5410
      NEMO5420
      NEMO5430
      NEMO5440
      NEMO5450
      NEMO5460
      NEMO5470
      NEMO5480
      NEMO5490
      NEMO5500
      NEMO5510
      NEMO5520
      NEMO5530
      NEMO5540
      NEMO5550
      NEMO5560
      NEMO5570
      NEMO5580
      NEMO5590
      NEMO5600
      NEMO5610
      NEMO5620
      NEMO5630
      NEMO5640
      NEMO5650
      NEMO5660
      NEMO5670
      NEMO5680
      NEMO5690
      NEMO5700
      NEMO5710
      NEMO5720
      NEMO5730
      NEMO5740
      NEMO5750
      NEMO5760

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ENDIF
IF(NATTN.EQ.40)THEN
  IF(ORDER.EQ.2)SR=5.76107
  IF(ORDER.EQ.3)SR=2.13923
  IF(ORDER.EQ.4)SR=1.40842
  IF(ORDER.EQ.5)SR=1.16811
  IF(ORDER.EQ.6)SR=1.07316
ENDIF
IF(NATTN.EQ.50)THEN
  IF(ORDER.EQ.2)SR=10.19181
  IF(ORDER.EQ.3)SR=3.04137
  IF(ORDER.EQ.4)SR=1.75285
  IF(ORDER.EQ.5)SR=1.33243
  IF(ORDER.EQ.6)SR=1.15868
ENDIF
IF(NATTN.EQ.60)THEN
  IF(ORDER.EQ.2)SR=18.09398
  IF(ORDER.EQ.3)SR=4.39729
  IF(ORDER.EQ.4)SR=2.24440
  IF(ORDER.EQ.5)SR=1.56860
  IF(ORDER.EQ.6)SR=1.28693
ENDIF
IF(NATTN.EQ.70)THEN
  IF(ORDER.EQ.2)SR=32.15951
  IF(ORDER.EQ.3)SR=6.40894
  IF(ORDER.EQ.4)SR=2.92363
  IF(ORDER.EQ.5)SR=1.88906
  IF(ORDER.EQ.6)SR=1.46330
ENDIF
ENDIF
IF(ASUBP.EQ.3.0)THEN
  IF(NATTN.EQ.20)THEN
    IF(ORDER.EQ.2)SR=1.73915
    IF(ORDER.EQ.3)SR=1.15516
    IF(ORDER.EQ.4)SR=1.03853
    IF(ORDER.EQ.5)SR=1.00996
    IF(ORDER.EQ.6)SR=1.00260
  ENDIF
  IF(NATTN.EQ.30)THEN
    IF(ORDER.EQ.2)SR=2.70352
    IF(ORDER.EQ.3)SR=1.45814
    IF(ORDER.EQ.4)SR=1.14542
    IF(ORDER.EQ.5)SR=1.05020
    IF(ORDER.EQ.6)SR=1.01783
  ENDIF
  IF(NATTN.EQ.40)THEN
    IF(ORDER.EQ.2)SR=5.05584
    IF(ORDER.EQ.3)SR=1.98022
    IF(ORDER.EQ.4)SR=1.34663
    IF(ORDER.EQ.5)SR=1.13934
    IF(ORDER.EQ.6)SR=1.05892
  ENDIF
  IF(NATTN.EQ.50)THEN
    IF(ORDER.EQ.2)SR=8.93009
    IF(ORDER.EQ.3)SR=2.79862
    IF(ORDER.EQ.4)SR=1.66148
    IF(ORDER.EQ.5)SR=1.28850
    IF(ORDER.EQ.6)SR=1.13534
  ENDIF
  IF(NATTN.EQ.60)THEN
    IF(ORDER.EQ.2)SR=15.84610
    IF(ORDER.EQ.3)SR=4.03471
    IF(ORDER.EQ.4)SR=2.11596
    IF(ORDER.EQ.5)SR=1.50711
    IF(ORDER.EQ.6)SR=1.25325
  ENDIF
  IF(NATTN.EQ.70)THEN
    IF(ORDER.EQ.2)SR=28.15950
    IF(ORDER.EQ.3)SR=5.87251
    IF(ORDER.EQ.4)SR=2.74749
    IF(ORDER.EQ.5)SR=1.80680
    IF(ORDER.EQ.6)SR=1.41799
  ENDIF
ENDIF
NEW05770
NEW05780
NEW05790
NEW05800
NEW05810
NEW05820
NEW05830
NEW05840
NEW05850
NEW05860
NEW05870
NEW05880
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NEW05900
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NEW06000
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NEW06100
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NEW06150
NEW06160
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NEW06180
NEW06190
NEW06200
NEW06210
NEW06220
NEW06230
NEW06240
NEW06250
NEW06260
NEW06270
NEW06280
NEW06290
NEW06300
NEW06310
NEW06320
NEW06330
NEW06340
NEW06350
NEW06360
NEW06370
NEW06380
NEW06390
NEW06400
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NEW06460
NEW06470
NEW06480

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      ENDIF
      ENDIF
      SPI=3.14159
      RK=1.0/SR
      N=ORDER
      EN=N
      KPR=SQRT(1.0-RK**2)
      Q0=0.5*(1.0-SQRT(KPR))/(1.0+SQRT(KPR))
      Q=Q0+2.0*Q0**5+15.0*Q0**9+150.0*Q0**13
      D=(10.0*(0.1*ASUBA)-1.0)/(10.0*(0.1*ASUBP)-1.0)
      LAM=(1.0/(2.0*EN))*ALOG((10.0*(0.05*ASUBP)+1.0)/(10.0*(0.05
C      *ASUBP)-1.0))
      ISTOP=0
      DENSUM=0.0
      ENSUM=SINH(LAM)
      M=1
17      EM=M
      ENUM=(((-1.0)**M)*(Q**(M*(M+1))))*SINH((2.0*EM+1.0)*LAM)
      DEN=2.0*(((-1.0)**M)*(Q**(M**2)))*COSH(2.0*EM*LAM)
      ENSUM=ENSUM+ENUM
      DENSUM=DENSUM+DEN
      IF(M.GE.2)THEN
        RN=ABS(ENUM/ENSUM)
        RD=ABS(DEN/(1.0+DENSUM))
        IF((RN.LE.1.0E-8).AND.(RD.LE.1.0E-8))THEN
          ISTOP=1
        ENDIF
      ENDIF
      M=M+1
      IF(ISTOP.EQ.0)THEN
        GO TO 17
      ENDIF
      SIGMA0=ABS((2.0*(Q**0.25)*ENSUM)/(1.0+DENSUM))
      W=SQRT((1.0+RK*(SIGMA0**2))*(1.0+(SIGMA0**2/RK)))
      IN=MOD(N,2)
      IF(IN.EQ.0)THEN
        IR=N/2
      ELSE
        IR=(N-1)/2
      ENDIF
      DO 18 I=1,IR
        IF(IN.EQ.0)THEN
          EMU=I-0.5
        ELSE
          EMU=I
        ENDIF
        ISTOP=0
        DENSUM=0.0
        ENSUM=SIN(PI*EMU/EN)
        M=1
19      EM=M
      ENUM=(((-1.0)**M)*(Q**(M*(M+1))))*SIN((2.0*EM+1.0)*PI*EMU/EN)
      DEN=2.0*(((-1.0)**M)*(Q**(M**2)))*COS(2.0*EM*PI*EMU/EN)
      ENSUM=ENSUM+ENUM
      DENSUM=DENSUM+DEN
      IF(M.GE.2)THEN
        RN=ABS(ENUM/ENSUM)
        RD=ABS(DEN/(1.0+DENSUM))
        IF((RN.LE.1.0E-8).AND.(RD.LE.1.0E-8))THEN
          ISTOP=1
        ENDIF
      ENDIF
      M=M+1
      IF(ISTOP.EQ.0)THEN
        GO TO 19
      ENDIF
      OMEGA(I)=(2.0*(Q**0.25)*ENSUM)/(1.0+DENSUM)
      V(I)=SQRT((1.0-RK*OMEGA(I)**2)*(1.0-(OMEGA(I)**2/RK)))
      AO(I)=1.0/(OMEGA(I)**2)
      DN=1.0+(SIGMA0*OMEGA(I))**2
      BO(I)=((SIGMA0*V(I))**2+(OMEGA(I)*W)**2)/(DN**2)
      BI(I)=(2.0*SIGMA0*V(I))/DN

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NEW06490
 NEW06500
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 NEW06580
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 NEW07000
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 NEW07100
 NEW07110
 NEW07120
 NEW07130
 NEW07140
 NEW07150
 NEW07160
 NEW07170
 NEW07180
 NEW07190
 NEW07200


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18      CONTINUE
      HO=1.0
      IF(IN.EQ.0)THEN
        DO 20 I=1,IR
          HO=HO*B0(I)/A0(I)
20      CONTINUE
      HO=HO*(10.0**(-0.05*ASUBP))
      ELSE
        DO 21 I=1,IR
          HO=HO*B0(I)/A0(I)
21      CONTINUE
      HO=HO*SIGMA0
      ENDIF
      R=DBLE(SR)
      HO=DBLE(HO)
      A01=DBLE(A0(1))
      B01=DBLE(B0(1))
      B11=DBLE(B1(1))
      IF((ORDER.EQ.2).OR.(ORDER.EQ.3))THEN
        APROT(1,0)=HO*A01
        APROT(1,1)=0.0D+00
        APROT(1,2)=HO
        APROT(1,3)=0.0D+00
        IF(ORDER.EQ.2)THEN
          APROT(2,0)=B01
          APROT(2,1)=B11
          APROT(2,2)=1.0D+00
        ENDIF
        IF(ORDER.EQ.3)THEN
          S0=DBLE(SIGMA0)
          APROT(2,0)=S0*B01
          APROT(2,1)=B01+(B11*S0)
          APROT(2,2)=S0+B11
          APROT(2,3)=1.0D+00
        ENDIF
      ENDIF
      IF((ORDER.EQ.4).OR.(ORDER.EQ.5))THEN
        A02=DBLE(A0(2))
        B02=DBLE(B0(2))
        B12=DBLE(B1(2))
        APROT(1,0)=HO*A01*A02
        APROT(1,1)=0.0D+00
        APROT(1,2)=HO*(A01+A02)
        APROT(1,3)=0.0D+00
        APROT(1,4)=HO
        APROT(1,5)=0.0D+00
        IF(ORDER.EQ.4)THEN
          APROT(2,0)=B01*B02
          APROT(2,1)=(B11*B02)+(B01*B12)
          APROT(2,2)=B01+B02+(B11*B12)
          APROT(2,3)=B11+B12
          APROT(2,4)=1.0D+00
        ENDIF
        IF(ORDER.EQ.5)THEN
          S0=DBLE(SIGMA0)
          APROT(2,0)=S0*B01*B02
          APROT(2,1)=(S0*((B11*B02)+(B01*B12)))+(B01*B02)
          APROT(2,2)=(S0*(B01+B02+(B11*B12)))+(B11*B02)+(B01*B12)
          APROT(2,3)=(S0*(B11+B12))+B01+B02+(B11*B12)
          APROT(2,4)=S0+B11+B12
          APROT(2,5)=1.0D+00
        ENDIF
      ENDIF
      IF((ORDER.EQ.6).OR.(ORDER.EQ.7))THEN
        A02=DBLE(A0(2))
        A03=DBLE(A0(3))
        B02=DBLE(B0(2))
        B03=DBLE(B0(3))
        B12=DBLE(B1(2))
        B13=DBLE(B1(3))
        APROT(1,0)=HO*A01*A02*A03
        APROT(1,1)=0.0D+00

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NEW07210
NEW07220
NEW07230
NEW07240
NEW07250
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NEW07610
NEW07620
NEW07630
NEW07640
NEW07650
NEW07660
NEW07670
NEW07680
NEW07690
NEW07700
NEW07710
NEW07720
NEW07730
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NEW07770
NEW07780
NEW07790
NEW07800
NEW07810
NEW07820
NEW07830
NEW07840
NEW07850
NEW07860
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NEW07900
NEW07910
NEW07920

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FILE: DFCADD FORTRAN A

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      APROT(1,2)=H0*((A02*A03)+(A01*(A02+A03)))
      APROT(1,3)=0.0D+00
      APROT(1,4)=H0*(A01+A02+A03)
      APROT(1,5)=0.0D+00
      APROT(1,6)=H0
      IF((ORDER.EQ.6).OR.(ORDER.EQ.7))THEN
        APROT(2,0)=B01*B02*B03
        APROT(2,1)=(B11*B02*B03)+(B01*((B02*B13)+(B12*B03)))
        APROT(2,2)=(B02*B03)+(B11*((B02*B13)+(B12*B03)))+(
          C      (B01*((B02+B03)+(B12*B13)))
          C      APROT(2,3)=B01*(B12+B13)+(B11*((B02+B03)+(B12*B13)))+(
            ((B02*B13)+(B12*B03))
            APROT(2,4)=B01+(B11*(B12+B13))+((B02+B03)+(B12*B13))
            APROT(2,5)=B11+B12+B13
            APROT(2,6)=1.0D+00
            ENDIF
            IF(ORDER.EQ.7)THEN
              S0=DBLE(SIGMA0)
              APROT(2,7)=1.0D+00
              APROT(2,6)=(S0*APROT(2,6))+APROT(2,5)
              APROT(2,5)=(S0*APROT(2,5))+APROT(2,4)
              APROT(2,4)=(S0*APROT(2,4))+APROT(2,3)
              APROT(2,3)=(S0*APROT(2,3))+APROT(2,2)
              APROT(2,2)=(S0*APROT(2,2))+APROT(2,1)
              APROT(2,1)=(S0*APROT(2,1))+APROT(2,0)
              APROT(2,0)=S0*APROT(2,0)
            ENDIF
            ENDIF
            ENDIF
            NORMALIZE THE ELLIPTIC PROTOTYPES
            IF(TYPE.EQ.5)THEN
              DO 25 I=1,2
              DO 26 J=0,ORDER
                APROT(I,J)=APROT(I,J)/((DSQRT(R))*J)
              26 CONTINUE
              25 CONTINUE
            ENDIF
            C APROT(TYPE=6) ALLOWS THE INPUT OF ANY TYPE OF PROTOTYPE FILTER
            C BY INPUTING THE COEFFICIENTS OF S
            IF(TYPE.EQ.6)THEN
              WRITE(6,*)
              WRITE(6,*) 'THIS OPTION ALLOWS YOU TO USE ANY ANALOG '
              WRITE(6,*) ' PROTOTYPE OF ORDER 1-10 '
              WRITE(6,*) '** COEFFICIENTS MUST BE IN DOUBLE PRECISION '
              WRITE(6,*) ' FORMAT (0.123456789D+00) ** '
              WRITE(6,*)
              WRITE(6,*) 'INPUT THE NUMERATOR COEFFICIENTS FIRST'
              DO 50 I=0,ORDER
                WRITE(6,*) 'WHAT IS THE COEFFICIENT OF S**',I,'?'
                50 READ(5,*) APROT(1,I)
                CONTINUE
                WRITE(6,*)
                CALL EXCMS ('CLRSCRN')
                WRITE(6,*) 'NOW INPUT THE DENOMINATOR COEFFICIENTS'
                WRITE(6,*)
                DO 51 J=0,ORDER
                  WRITE(6,*) 'WHAT IS THE COEFFICIENT OF S**',J,'?'
                  51 READ(5,*) APROT(2,J)
                  CONTINUE
                ENDIF
                C *****
                C *****
                C DEFINE ELEMENTS OF THE BILINEAR TRANSFORMATION MATRIX
                C *****
                C FOR ALL THE BILINEAR TRANSFORMATION MATRICES, THE FIRST COLUMN
                C HAS ALTERNATING +1'S AND -1'S. THE LAST COLUMN HAS ALL +1'S
                DO 60 I=1,ORDER
                  DO 61 J=0,I
                    BILINR(I,J,0)=(-1.0)**J
                    BILINR(I,J,I)=1.0
                  61 CONTINUE
                  60 CONTINUE

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FILE: DFCADD FORTRAN A

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C FIRST ORDER MATRIX HAS ALREADY BEEN LOADED CONTINUE INTERATION NEW08650
C UNTIL YOU HAVE THE APPROPRIATE MATRIX NEW08660
C DO 75 K=2,ORDER NEW08670
  KM1=K-1 NEW08680
C LOAD ELEMENTS IN INTERIOR COLUMNS, ALL ROWS EXCEPT LAST ONE NEW08690
  DO 76 L=0,KM1 NEW08700
    DO 77 M=1,KM1 NEW08710
      MM1=M-1 NEW08720
      BILINR(K,L,M)=BILINR(KM1,L,M)+BILINR(KM1,L,MM1) NEW08730
77 CONTINUE NEW08740
76 CONTINUE NEW08750
C NOW LOAD THE LAST ROW NEW08760
  DO 78 N=1,KM1 NEW08770
    BILINR(K,K,N)=BILINR(K,0,N)*((-1.0D+00)**(K+N)) NEW08780
78 CONTINUE NEW08790
75 CONTINUE NEW08800
C ***** NEW08810
C THE FOLLOWING CLEARS DPROT PRIOR TO LOADING NEW08820
C DO 80 K=1,2 NEW08830
  DO 81 L=0,10 NEW08840
    DPROT(K,L)=0.0 NEW08850
81 CONTINUE NEW08860
80 CONTINUE NEW08870
C ***** NEW08880
C DIGITIZE THE ANALOG PROTOTYPE USING THE BILINEAR TRANSFORMATION NEW08890
C ***** NEW08900
C DO 100 N=1,2 NEW08910
  DO 101 J=0,ORDER NEW08920
    SUMR=0.0 NEW08930
    DO 102 I=0,ORDER NEW08940
      TEMP=APROT(N,I)*BILINR(ORDER,I,J) NEW08950
      SUMR=SUMR+TEMP NEW08960
102 CONTINUE NEW08970
      DPROT(N,J)=SUMR NEW08980
101 CONTINUE NEW08990
100 CONTINUE NEW09000
C ***** NEW09010
C OPTION TO SHOW DIGITAL PROTOTYPE COEFFICIENTS NEW09020
C CALL EXCMS ('CLRSCRN') NEW09030
  WRITE(6,*) 'THE DIGITAL PROTOTYPE HAS BEEN COMPUTED. DO YOU' NEW09040
  WRITE(6,*) 'WISH TO CHECK THE PROTOTYPE COEFFICIENTS?' NEW09050
  WRITE(6,*) ' ** 1-YES/2-NO **' NEW09060
  READ(5,*) LOOK NEW09070
  IF(LOOK.EQ.1)THEN NEW09080
C MAKE DENOMINATOR COEFFICIENT OF Z**ORDER = 1.0 NEW09090
  DO 105 K=1,2 NEW09100
    DO 106 L=0,ORDER NEW09110
      DPROT(K,L)=DPROT(K,L)/DPROT(2,ORDER) NEW09120
106 CONTINUE NEW09130
105 CONTINUE NEW09140
  CALL EXCMS ('CLRSCRN') NEW09150
  WRITE(6,*) 'THE DIGITAL PROTOTYPE NUMERATOR COEFFICIENTS ARE:' NEW09160
  DO 111 N=ORDER,0,-1 NEW09170
    WRITE(6,902) DPROT(1,N),N NEW09180
111 CONTINUE NEW09190
    WRITE(6,*) NEW09200
    WRITE(6,*) 'THE DIGITAL PROTOTYPE DENOMINATOR COEFFICIENTS ARE:' NEW09210
    DO 112 M=ORDER,0,-1 NEW09220
      WRITE(6,902) DPROT(2,M),M NEW09230
112 CONTINUE NEW09240
      WRITE(6,*) NEW09250
      WRITE(6,*) '** 1 TO CONTINUE **' NEW09260
      READ(5,*) LOOK NEW09270
    ENDIF NEW09280
C ***** NEW09290
C NOW DO THE DIGITAL TO DIGITAL TRANSFORMATIONS NEW09300
C ***** NEW09310
C IF((PASS.EQ.1).OR.(PASS.EQ.2))THEN NEW09320
C DEFINE ELEMENTS OF THE LOWPASS AND HIGHPASS DIGITAL-DIGITAL NEW09330
C TRANSFORMATION MATRIX NEW09340
C DEFINE THE ELEMENTS FOR FIRST AND LAST COLUMNS NEW09350
  NEW09360

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FILE: DFCADD FORTRAN A

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      DO 170 I=1,10
      DO 171 J=0,I
        LPHPTR(I,J,0)=((-1.0)*ALPHA)**J
        IMJ=I-J
        LPHPTR(I,IMJ,I)=LPHPTR(I,J,0)
171    CONTINUE
170    CONTINUE
C      FIRST ORDER MATRIX HAS BEEN DEFINED ABOVE
C      SECOND TO TENTH ORDER BELOW
      DO 175 K=2,10
        KM1=K-1
        DO 176 L=0,KM1
          DO 177 M=1,KM1
            MM1=M-1
            LPHPTR(K,L,M)=LPHPTR(KM1,L,M)+(LPHPTR(KM1,L,MM1))*(-1.0D+00)
C*ALPHA)
177    CONTINUE
176    CONTINUE
          DO 178 N=1,KM1
            KMN=K-N
            LPHPTR(K,K,N)=LPHPTR(K,0,KMN)
178    CONTINUE
175    CONTINUE
C      *****
C      CALCULATE THE FINAL FILTER
C      FOR HIGHPASS, THE PROTOTYPE MUST SUBSTITUTE (-Z) FOR (Z)
      IF(PASS.EQ.2)THEN
        DO 180 K=1,2
          DO 181 L=0,ORDER
            DPROT(K,L)=DPROT(K,L)*((-1.0)**L)
181    CONTINUE
180    CONTINUE
          ENDIF
          DO 185 K=1,2
            DO 186 L=0,ORDER
              SUMR=0.0D+00
              DO 187 M=0,ORDER
                TEMP=DPROT(K,M)*LPHPTR(ORDER,M,L)
                SUMR=SUMR+TEMP
187    CONTINUE
              DFLTR(K,L)=SUMR
186    CONTINUE
185    CONTINUE
          ENDIF
C      *****
C      BANDPASS AND BANDSTOP FORMULAS
      IF((PASS.EQ.3).OR.(PASS.EQ.4))THEN
C      LOAD THE BANDPASS/BANDSTOP TRANSFORMATION MATRIX
C      FIRST LOAD THE FIRST AND LAST COLUMNS
        DO 225 I=1,10
          DO 226 J=0,I
            BPBSTR(I,J,0)=C**J
            I2=2*I
            IMJ=I-J
            BPBSTR(I,IMJ,I2)=BPBSTR(I,J,0)
226    CONTINUE
225    CONTINUE
C      LOAD THE REST OF THE FIRST ORDER MATRIX
        BPBSTR(1,0,1)=(-1.0D+00)*B
        BPBSTR(1,1,1)=(-1.0D+00)*B
C      LOAD THE 2ND-10TH ORDER TRANSFORMATION MATRICES
        DO 230 K=2,10
          K2=2*K
          KM1=K2-1
          KM1=K-1
          K2M2=K2-2
          KM12=2*KM1
          KM12M1=KM12-1
C      LOAD THE SECOND AND SECOND TO LAST COLUMNS
          DO 231 L=0,KM1
            BPBSTR(K,L,1)=BPBSTR(KM1,L,1)+(BPBSTR(KM1,L,0)*(-1.0D+00)*B)
            BPBSTR(K,0,K2M1)=(BPBSTR(KM1,0,KM12)*(-1.0D+00)*B)+

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FILE: DFCADD FORTRAN A

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C      (BPBSTR(KM1,0,KM1ZM1)*C) NEW10090
      KML=K-L NEW10100
      BPBSTR(K,KML,K2M1)=BPBSTR(K,L,1) NEW10110
231  CONTINUE NEW10120
C      NOW LOAD INTERIOR ELEMENTS EXCEPT FOR LAST ROW NEW10130
      DO 236 L=0,KM1 NEW10140
      DO 237 M=2,K2M2 NEW10150
      MM1=M-1 NEW10160
      MM2=M-2 NEW10170
      BPBSTR(K,L,M)=BPBSTR(KM1,L,M)+(BPBSTR(KM1,L,MM1)*(-1.0D+00)* NEW10180
      B)+(BPBSTR(KM1,L,MM2)*C) NEW10190
237  C CONTINUE NEW10200
236  CONTINUE NEW10210
C      NOW FILL THE LAST ROW NEW10220
      DO 238 N=1,K2M1 NEW10230
      K2MN=K2-N NEW10240
      BPBSTR(K,K,K2MN)=BPBSTR(K,0,N) NEW10250
238  CONTINUE NEW10260
230  CONTINUE NEW10270
C      ***** NEW10280
C      CALCULATE THE FINAL FILTER NEW10290
C      FOR THE BANDPASS CASE REPLACE (Z) WITH (-Z) NEW10300
      IF(PASS.EQ.3)THEN NEW10310
      DO 240 M=1,2 NEW10320
      DO 241 N=0,ORDER NEW10330
      DPROT(M,N)=DPROT(M,N)*((-1.0D+00)**N) NEW10340
241  CONTINUE NEW10350
240  CONTINUE NEW10360
      ENDIF NEW10370
      ORDER2=2*ORDER NEW10380
      DO 245 K=1,2 NEW10390
      DO 246 L=0,ORDER2 NEW10400
      SUMR=0.0D+00 NEW10410
      DO 247 M=0,ORDER NEW10420
      TEMP=DPROT(K,M)*BPBSTR(ORDER,M,L) NEW10430
      SUMR=SUMR+TEMP NEW10440
247  CONTINUE NEW10450
      DFLTR(K,L)=SUMR NEW10460
246  CONTINUE NEW10470
245  CONTINUE NEW10480
      ENDIF NEW10490
C      ***** NEW10500
C      OUTPUT OPTIONS NEW10510
C      ***** NEW10520
      IF((PASS.EQ.3).OR.(PASS.EQ.4))THEN NEW10530
      END=2*ORDER NEW10540
      ENDIF NEW10550
      IF((PASS.EQ.1).OR.(PASS.EQ.2))THEN NEW10560
      END=ORDER NEW10570
      ENDIF NEW10580
C      MAKE DENOMINATOR COEFFICIENT OF Z**END EQUAL TO 1 NEW10590
      DO 250 M=1,2 NEW10600
      DO 251 N=0,END NEW10610
      DFLTR(M,N)=DFLTR(M,N)/DFLTR(2,END) NEW10620
251  CONTINUE NEW10630
250  CONTINUE NEW10640
      CALL EXCMS ('CLRSCRN') NEW10650
      WRITE(6,*) NEW10660
      WRITE(6,*) 'THE FINAL DIGITAL FILTER IS:' NEW10670
      WRITE(6,*) NEW10680
      WRITE(6,*) '    NUMERATOR COEFFICIENTS:' NEW10690
      DO 300 I=END,0,-1 NEW10700
      WRITE(6,902) DFLTR(1,I),I NEW10710
300  CONTINUE NEW10720
      WRITE(6,*) '    DENOMINATOR COEFFICIENTS:' NEW10730
      DO 301 J=END,0,-1 NEW10740
      WRITE(6,902) DFLTR(2,J),J NEW10750
301  CONTINUE NEW10760
      WRITE(6,*) NEW10770
      WRITE(6,*) ' ** ENTER 1 TO CONTINUE **' NEW10780
      READ(5,*) LOOK NEW10790
C      ***** NEW10800

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FILE: DFCADD FORTRAN A

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C      OPTION TO SPOT CHECK FILTER OUTPUT MAGNITUDE AND FREQUENCY
CALL EXCMS ('CLRSCRN')
WRITE(6,*) 'THE TRANSFER FUNCTION FOR THIS FILTER HAS BEEN'
WRITE(6,*) 'COMPUTED. DO YOU WANT TO SPOT CHECK THE MAGNITUDE'
WRITE(6,*) 'AND PHASE OF THE FILTER OUTPUT FOR A GIVEN '
WRITE(6,*) 'FREQUENCY?'
WRITE(6,*) '      ** 1-YES/2-NO ** '
READ(5,*) LOOK
IF(LOOK.EQ.1)THEN
310  CALL EXCMS ('CLRSCRN')
      IF(AFORDF.EQ.1)THEN
          WRITE(6,*) 'WHAT ANALOG FREQUENCY(IN KHZ)'
          WRITE(6,*) ' DO YOU WANT TO CHECK?'
          WRITE(6,*) '      ** DECIMAL **'
          READ(5,*) F
          DF=(F/SSFREQ)*2.0*SPI
      ENDIF
      IF(AFORDF.EQ.2)THEN
          WRITE(6,*) 'WHAT DIGITAL FREQUENCY DO YOU WANT TO CHECK?'
          WRITE(6,*) '      ** DECIMAL **'
          READ(5,*) DF
      ENDIF
      X=COS(DF)
      Y=SIN(DF)
      Z=CMPLX(X,Y)
      XN=REAL(DFLTR(1,0))
      YN=0.0
      XD=REAL(DFLTR(2,0))
      YD=0.0
      HOFZN=CMPLX(XN,YN)
      HOFZD=CMPLX(XD,YD)
      DO 320 N=1,END
          ZN=REAL(DFLTR(1,N))*(Z**N)
          ZD=REAL(DFLTR(2,N))*(Z**N)
          HOFZN=HOFZN+ZN
          HOFZD=HOFZD+ZD
320  CONTINUE
      HOFZ=HOFZN/HOFZD
      FMAG=SQRT((REAL(HOFZ)**2)+(AIMAG(HOFZ)**2))
      FMAGDB=20.0*ALOG10(FMAG)
      FPHSE=ATAN(AIMAG(HOFZ)/REAL(HOFZ))
      WRITE(6,*) 'THE FILTER OUTPUT WILL BE:'
      WRITE(6,914) FMAGDB
      WRITE(6,915) FPHSE
      WRITE(6,*)
      WRITE(6,*) 'DO YOU WANT TO CHECK ANOTHER FREQUENCY?'
      WRITE(6,*) '      ** 1-YES/2-NO ** '
      READ(5,*) LOOK
      IF(LOOK.EQ.1)GO TO 310
      ENDIF
C      *****
C      OPTION TO LOAD FREQUENCY RESPONSE INTO A DATA FILE
CALL EXCMS ('CLRSCRN')
WRITE(6,*) 'DO YOU WANT THE FILTERS FREQUENCY RESPONSE '
WRITE(6,*) 'DATA TO BE ENTERED INTO A DATA FILE FOR USE'
WRITE(6,*) 'BY A PLOTTING PROGRAM?'
WRITE(6,*) ' NOTE: THE FILE NAME WILL BE FILTER, FILE TYPE'
WRITE(6,*) '      WILL BE DATA, 100 POINTS WILL BE CALCULATED'
WRITE(6,*) '      FOR A DIGITAL FREQUENCY FROM 0 TO PI, THE FIRST'
WRITE(6,*) '      COLUMN WILL BE THE DIGITAL FREQUENCY (EXPRESSED '
WRITE(6,*) '      AS A FRACTION OF THE SAMPLING FREQUENCY 0-1/2) '
WRITE(6,*) '      THE SECOND COLUMN WILL BE THE MAGNITUDE, AND'
WRITE(6,*) '      THE THIRD COLUMN WILL BE THE PHASE(DEGREES)'
WRITE(6,*) '      ** 1-MAKE DATA FILE/2-DON'T BOTHER **'
READ(5,*) LOOK
IF(LOOK.EQ.1)THEN
      DO 350 I=0,100
          DF=REAL(I)*(5.0/1000.0)
          DFR=REAL(I)*(SPI/100.0)
          X=COS(DFR)
          Y=SIN(DFR)
          Z=CMPLX(X,Y)
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FILE: DFCADD FORTRAN A

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      XN=REAL(DFLTR(1,0))
      YN=0.0
      XD=REAL(DFLTR(2,0))
      YD=0.0
      HOFZN=CMPLX(XN,YN)
      HOFZD=CMPLX(XD,YD)
      DO 351 N=1,END
        ZN=REAL(DFLTR(1,N))*(Z**N)
        ZD=REAL(DFLTR(2,N))*(Z**N)
        HOFZN=HOFZN+ZN
        HOFZD=HOFZD+ZD
351    CONTINUE
      HOFZ=HOFZN/HOFZD
      FMAG=SQRT((REAL(HOFZ)**2)+(AIMAG(HOFZ)**2))
      IF(REAL(HOFZ).NE.0.0)THEN
        FPHSER=ATAN(AIMAG(HOFZ)/REAL(HOFZ))
      ELSEIF(REAL(HOFZ).EQ.0.0)THEN
        IF(AIMAG(HOFZ).GT.0.0)FPHSER=1.570796
        IF(AIMAG(HOFZ).LT.0.0)FPHSER=-1.570796
        IF(AIMAG(HOFZ).EQ.0.0)FPHSER=0.0
      ENDIF
      FPHSE=FPHSER*(180.0/SPI)
      WRITE(8,916) DF,FMAG,FPHSE
350    CONTINUE
      ENDIF
C      *****
901    FORMAT(5X,F10.5,' S**',I1)
902    FORMAT(5X,F10.5,' Z**',I1)
903    FORMAT(3X,'ALPHA= ',F9.4)
904    FORMAT(3X,'K= ',F9.4)
905    FORMAT(3X,'THE LOWPASS DIGITAL PROTOTYPE FREQ. IS ',F5.3)
914    FORMAT(3X,'MAGNITUDE = ',F8.2,' DB')
915    FORMAT(3X,'PHASE = ',F6.2,' RADIANS')
916    FORMAT(3X,F4.3,10X,F5.3,10X,F6.2)
      STOP
      END

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